Radar Systems Engineering
Lecture 15
Parameter Estimation
And Tracking
Part 1

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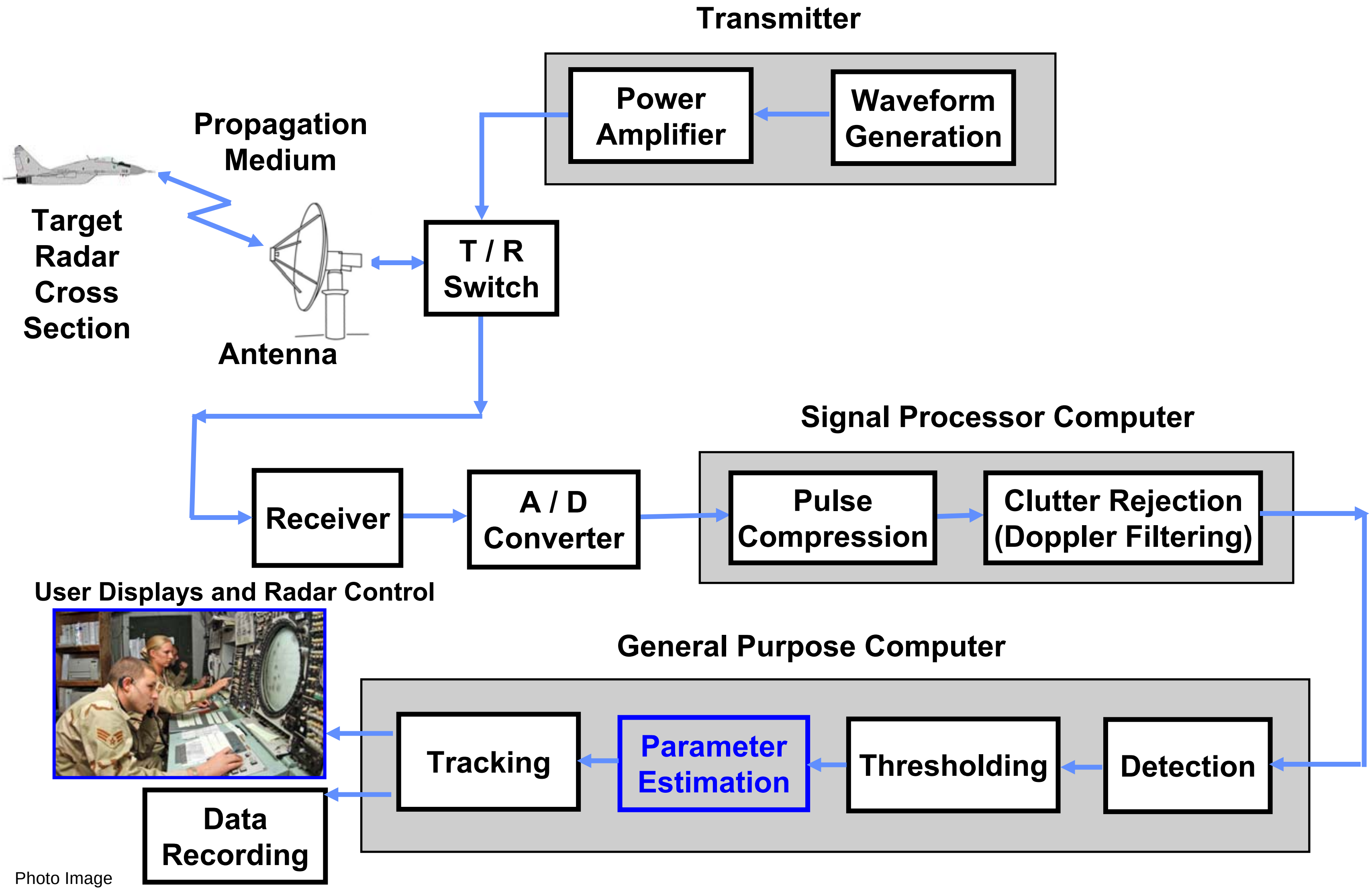
Guest Lecturer

Block Diagram of Radar System



Transmitter

Power Amplifier

Waveform Generation

Propagation Medium

Target Radar Cross Section

Antenna

T / R Switch

Signal Processor Computer

Receiver

A / D Converter

Pulse Compression

Clutter Rejection (Doppler Filtering)

User Displays and Radar Control

General Purpose Computer

Tracking

Parameter Estimation

Thresholding

Detection

Data Recording

Photo Image
Courtesy of US Air Force

Tracking Radars

Courtesy of Lockheed Martin.
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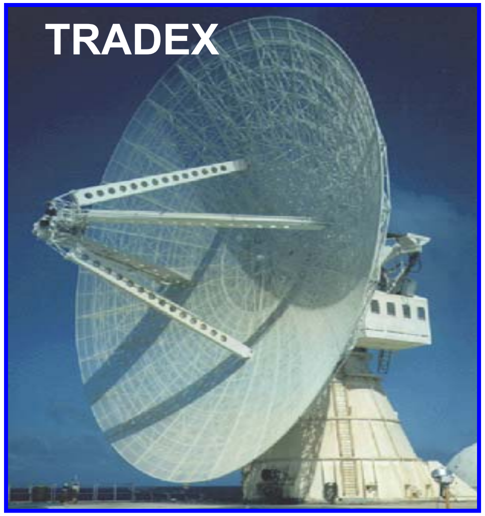
Courtesy of Raytheon, Used with Permission

Courtesy of US Air Force

Courtesy of FAA



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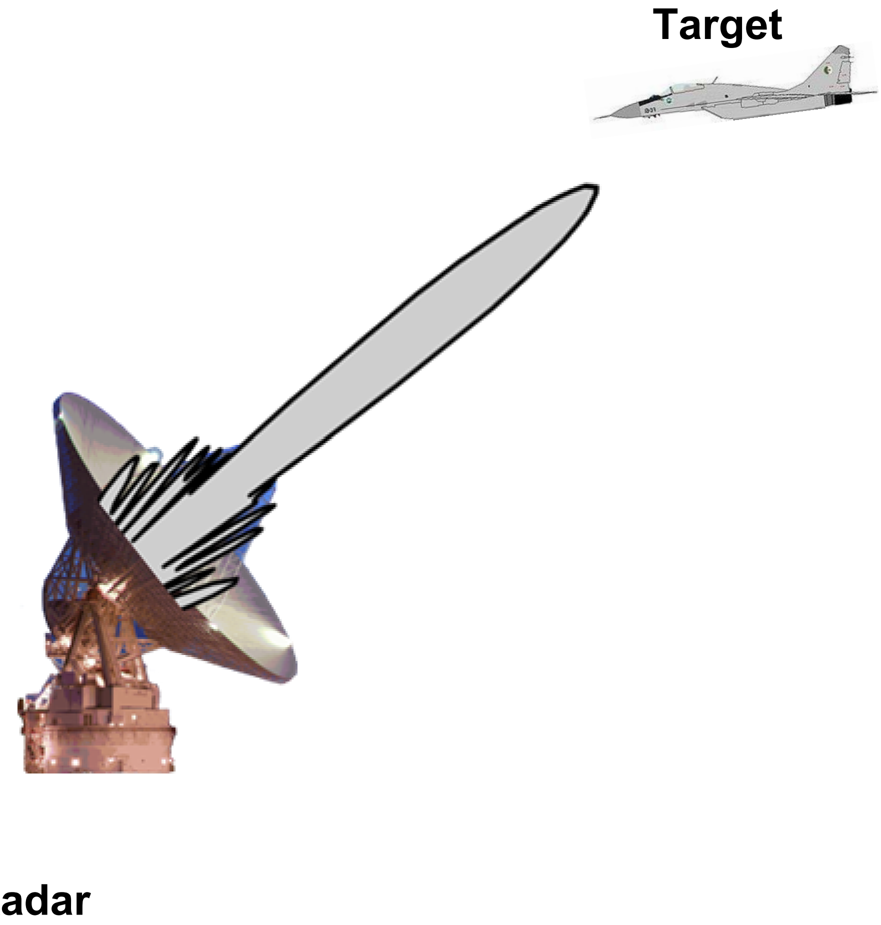
Outline

- Introduction
- Observable Estimation
- Single Target Tracking
- Multiple Target Tracking
- Summary

Radar Parameter Estimation



Measured Radar Observables

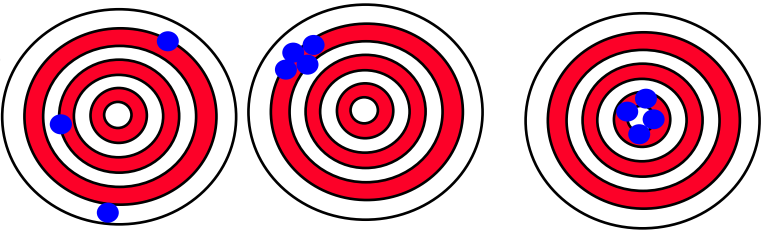
- Location
  - Range
  - Azimuth Angle
  - Elevation Angle
- Size
  - Amplitude (RCS)
  - Radial Extent
  - Cross Range Extent
- Motion
  - Radial Velocity (Doppler)
  - Acceleration
  - Angular Motion about Center of Mass
  - Ballistic Coefficient

Quantities in Blue Are Usually Measured Directly

Accuracy, Precision and Resolution

- **Precision:**
  - Repeatability of a measurement
- **Accuracy:**
  - The degree of conformity of measurement to the true value
  - Bias Error : True value- Average measured value
- **Resolution:**
  - Offset (angle or range) required for two targets to be recognized as separate targets

Example
Accuracy vs. Precision

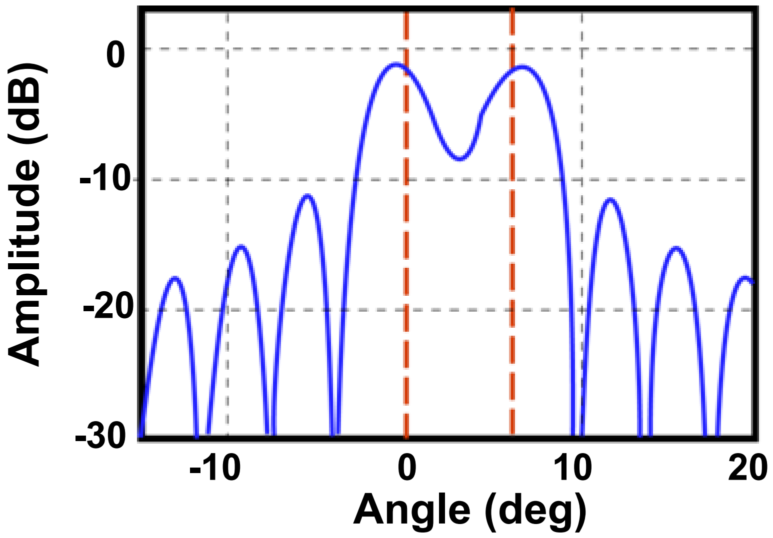


Low Accuracy
Low Precision

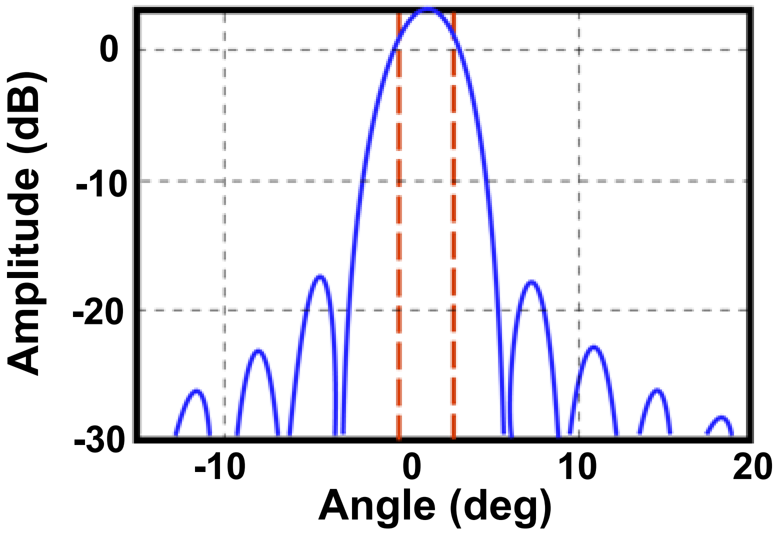
Low Accuracy
High Precision

High Accuracy
High Precision

Targets at 0° and 6°



Targets at 0° and 3°



Outline

- Introduction
- Observable Estimation
  - Range
  - Angle
  - Doppler
  - Amplitude of reflected echo from target
- Single Target Tracking
- Multiple Target Tracking
- Summary

Observable Accuracy

- Observable to be discussed
  - Range
  - Angle
  - Doppler Velocity

- After bias errors are accounted for, noise is the key limiting factor in accurately measuring the above observables
  - The exception is angle measurement, where for low angle tracking multipath errors can predominate

- The theoretical rms error δM of a measurement M is of the form

$$\delta M = \frac{k\,M}{\sqrt{S/N}}$$

  - Where k is a constant between .5 and 1

Limitations on Range Estimation

- Estimation of the range of a target is based upon using A/D sampled measurements of the round trip time to and from the target

$$R = \frac{c T_R}{2}$$

- For time delay measurements , such as range, the value of the constant k depends on the shape of the radar pulse's spectrum and the pulse's rise time.

- For a rectangular pulse, whose width is T $\quad \delta T \approx \frac{T}{2\sqrt{S/N}}$
  - Which yields $\quad \delta R = \frac{c T}{2\sqrt{S/N}}$

- For a train of pulses it becomes:

$$\delta R = \frac{c T}{2\sqrt{(S/N)(PRF)T_D}}$$

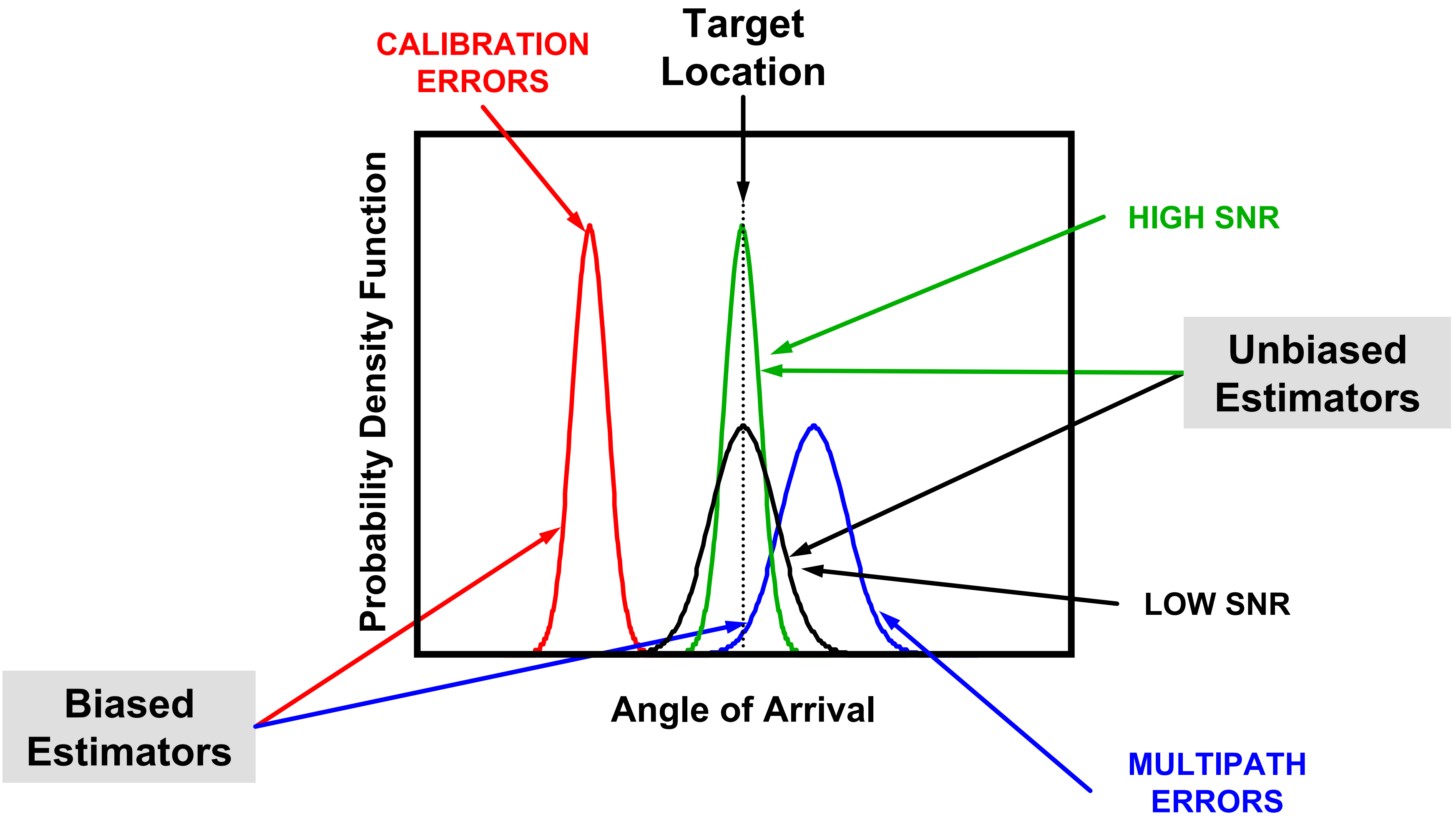
T_D = Dwell Time

Adapted from Barton and Ward
Reference 6

Theoretical vs. Practical Accuracy Limitations

- General
  - Section 6.3 of Skolnik reference 1 derives the theoretical limitations for each of the pertinent observables
    - Time, frequency, and angle
- Range
  - S/N, pulse shape and width, effective bandwidth, number of pulses
- Doppler Frequency
  - S/N, pulse shape, integration time
- Angle
  - S/N, type of measurement technique, antenna illumination distribution, antenna size, frequency

Angle Estimation Issues



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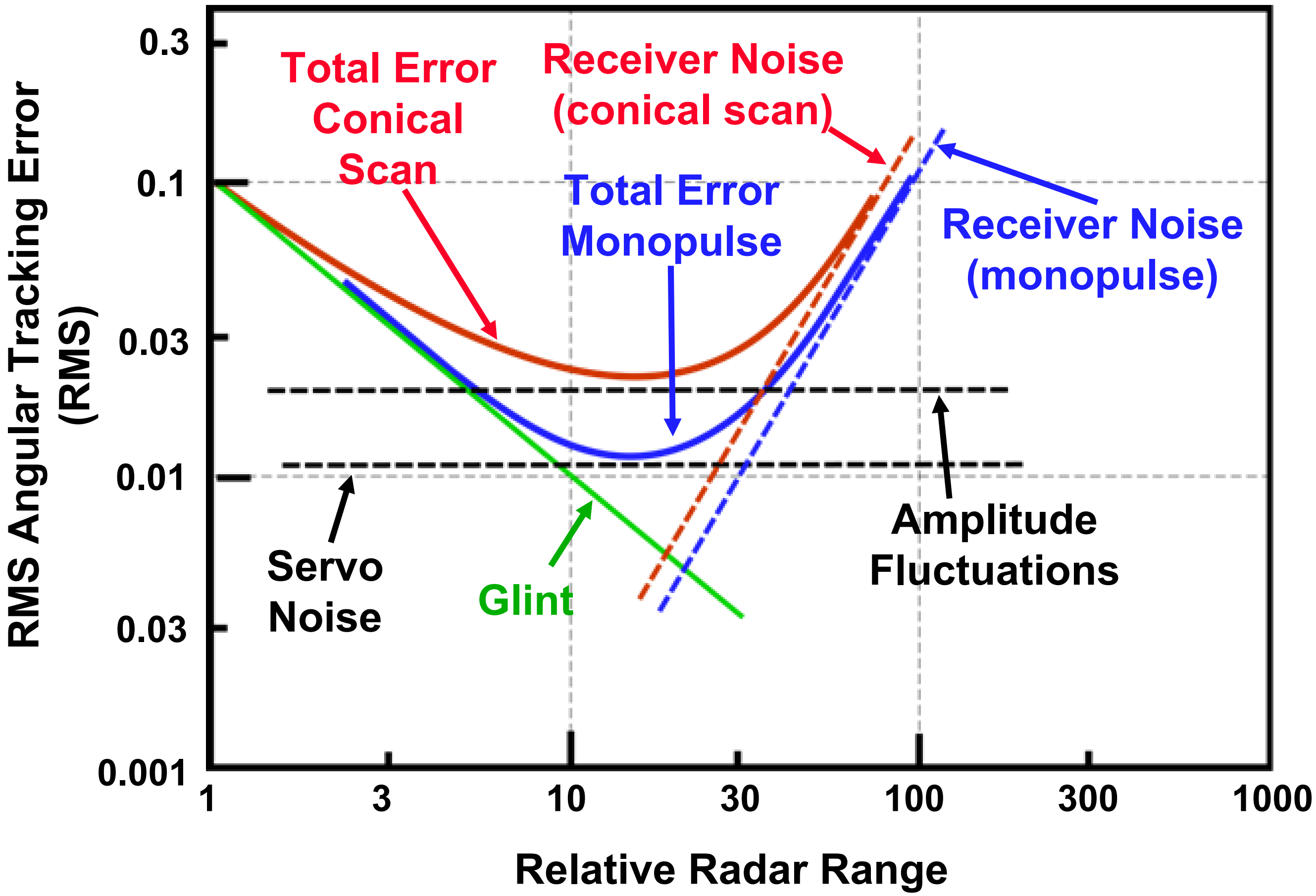
Limitation on Angle Estimation



Adapted from Skolnik
Reference 1

Sources of Error

Signal to Noise Ratio

Monopulse vs. Conical Scan

Servo Noise

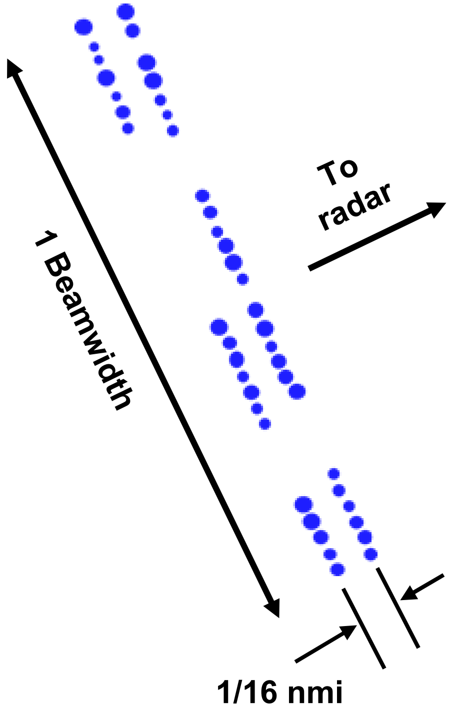
Amplitude Fluctuations

$$\delta\theta \approx \frac{.7\,\theta_{3DB}}{\sqrt{S/N}}$$

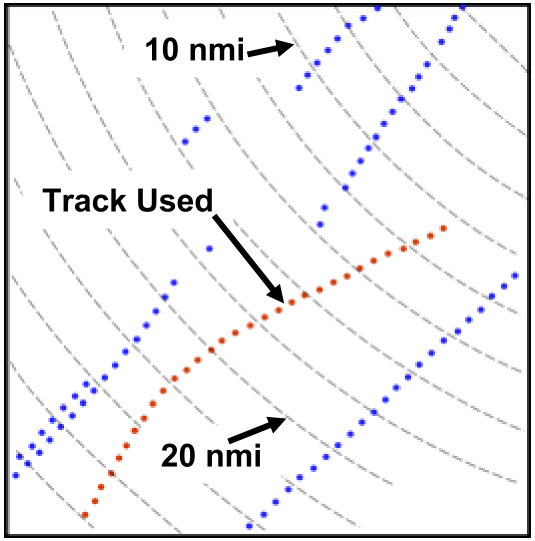
Angular Accuracy with ASR Radar

- Angular beam splitting with Track While Scan Radar
  - ~10 : 1 splitting measured

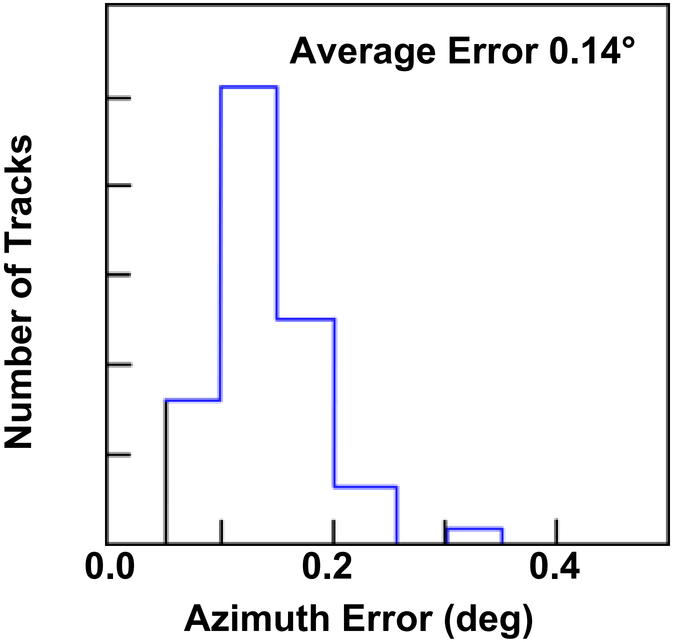
Target Detections From 4 CPI's



Sample Tracker Output



Accuracy of 100 tracks

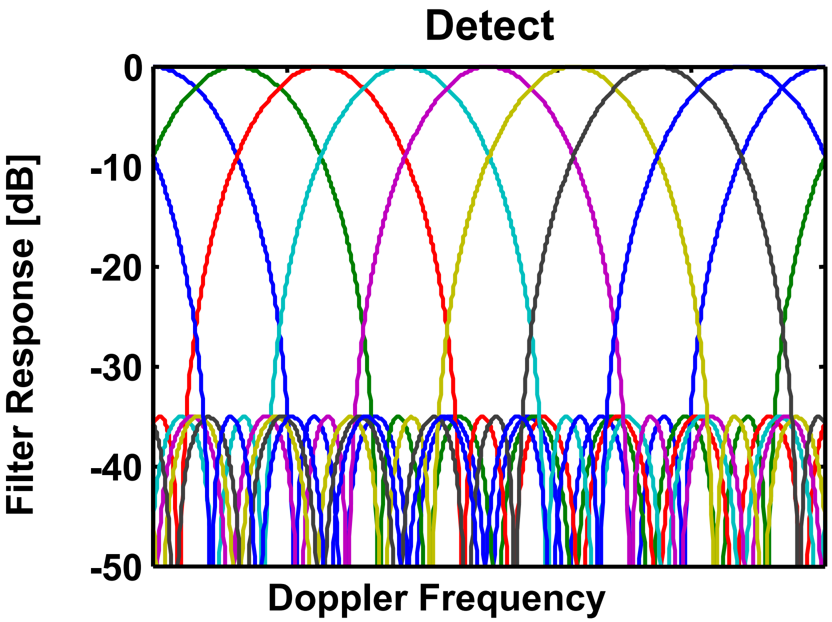


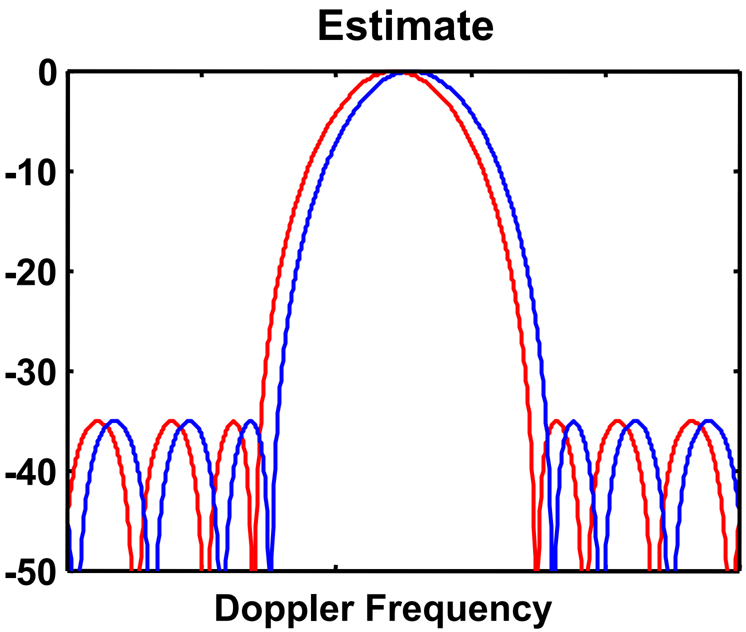
Doppler Estimation

Doppler Frequency → $f_d = \dfrac{2v_r}{\lambda}$

$2v_r$ ← Radial Velocity

λ ← Wavelength





- Filter-bank spans entire radar system Doppler frequency band
- Detections are isolated within a single Doppler filter

Courtesy of MIT Lincoln Laboratory, Used with Permission

- Use two closely spaced frequency filters offset from the center frequency of the Doppler filter containing the detection
- Doppler estimation procedure is similar to angle estimation with angle and frequency interchanged

Radar Cross Section Measurement Accuracy

- Measurement of the radar cross section (RCS) of a target in a test environment was discussed in detail in the lecture on Radar Cross Section (Lecture10)

- When one wants to measure the RCS of a target, the radar needs to be calibrated
  - How do A/D counts relate to RCS values?

- This calibration process is usually accomplished by launching a balloon with a sphere (RCS independent of orientation) attached by a lengthy tether and measuring the amplitude in A/D counts and the range of the balloon

Radar Cross Section Measurement Accuracy

- The calibration process (continued)
  - Measurement is performed in the far field
  - A radiosonde is usually balloon launched separately to measure the pressure, temperature, etc. (index of refraction of the atmosphere vs. height) so that propagation effects, such as, ducting, multipath, etc., may be taken into account properly and accurately
- High power radars could use spherical satellites to perform the same function as the balloon borne sphere
- RCS accuracy is usually limited by the ability to measure atmospheric (properties) losses as a function of the sphere’s range and elevation angle

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- → Single Target Tracking
  - – Angle tracking techniques
    - Amplitude monopulse
    - Phase comparison monopulse
    - Sequential lobing
    - Conical scanning
  - – Range tracking
  - – Servo systems
- Multiple Target Tracking
- Summary

TRADEX

Courtesy of MIT Lincoln Laboratory. Used with permission

FPS-16

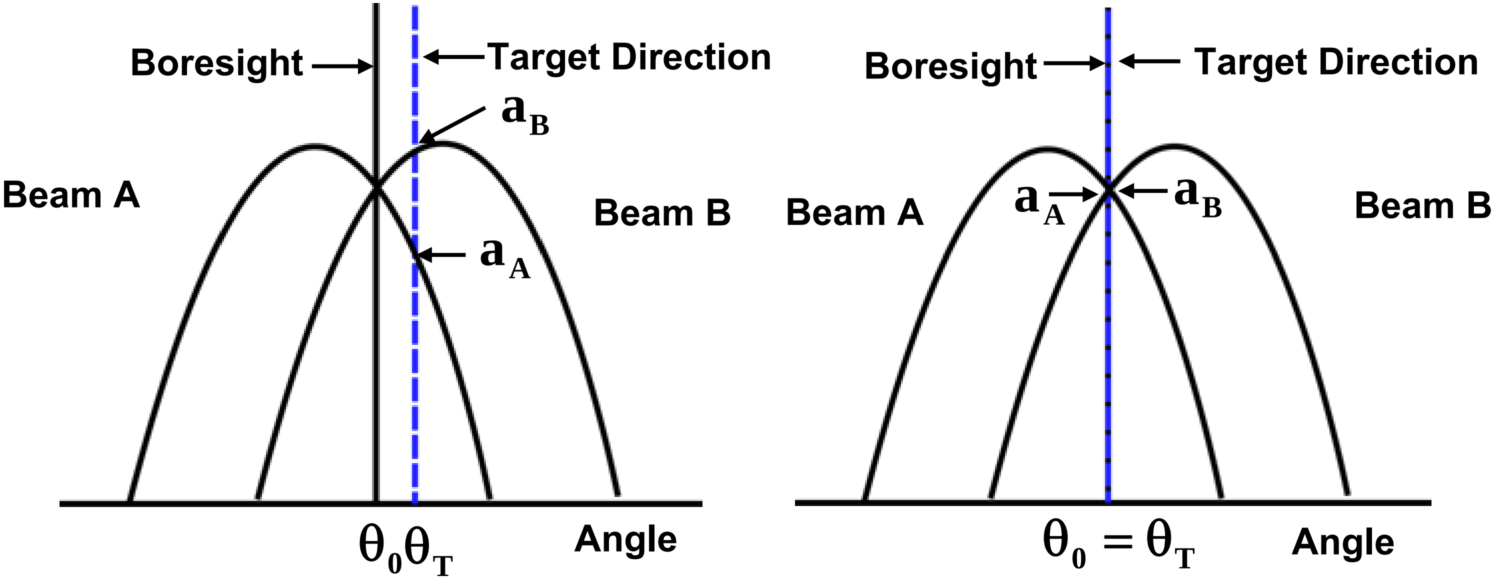
Courtesy of US Air Force

Single Target Tracking - General

- Usually after a target is initially detected, the radar is asked to:
  - Continue to detect the target as it moves through the radar's coverage
  - Associate the different detections with the specific target
    - "All these detections are from the same target"
    - Use range, angle, Doppler measurements
  - Use these detections to develop a continually more accurate estimate of the targets observables
    - Position, velocity, etc
  - Predict where the target will be is the future

- These are the functions of a "Tracker"

Basics of Continuous Angle Tracking



- For radars with a dish antenna, the purpose of the tracking function is to keep the antenna beam axis aligned with a selected target.
- Illustration at left
  - Two overlapping beams - target is to the right of antenna boresight $a_A < a_B$
- Illustration at right
  - Two overlapping beams - target is to the right of antenna boresight $a_A = a_B$. Target is located at boresight position.

Adapted from Skolnik
Reference 1

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    - ➡ Amplitude monopulse
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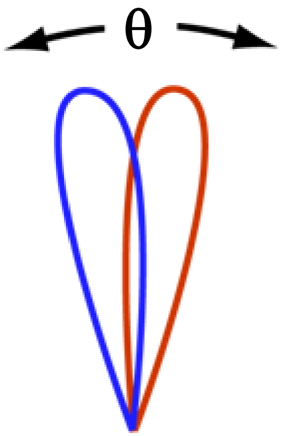
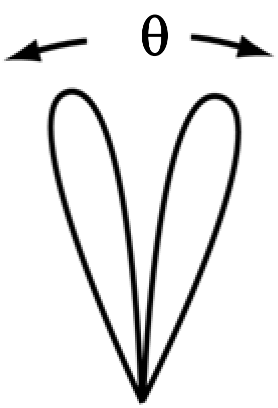
Amplitude Comparison Monopulse

- Amplitude Comparison Monopulse Method:
  - Use pairs of slightly offset beams to determine the location of the target relative to the antenna boresight (error signal)
  - Use this information to re-steer the antenna (or beam) to keep the target very close to the antenna boresight
- For dish antennas, two offset receive beams are generated by using two feeds slightly displaced in opposite directions from the focus of a parabolic reflector
- The sum and difference of the two squinted beams are used to generate the error signal
- Each channel (sum, azimuth difference, and elevation difference) requires a separate receiver

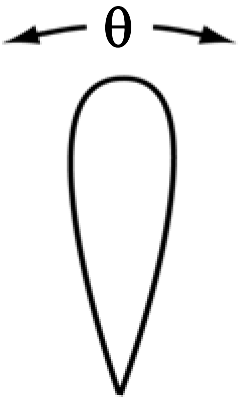
Monopulse Antenna Patterns and Error Signals



Overlapping Antenna Patterns

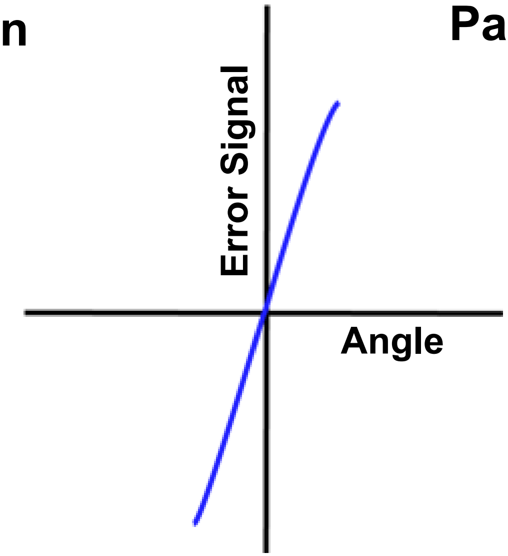


Difference Pattern
Δ



Sum Pattern
Σ

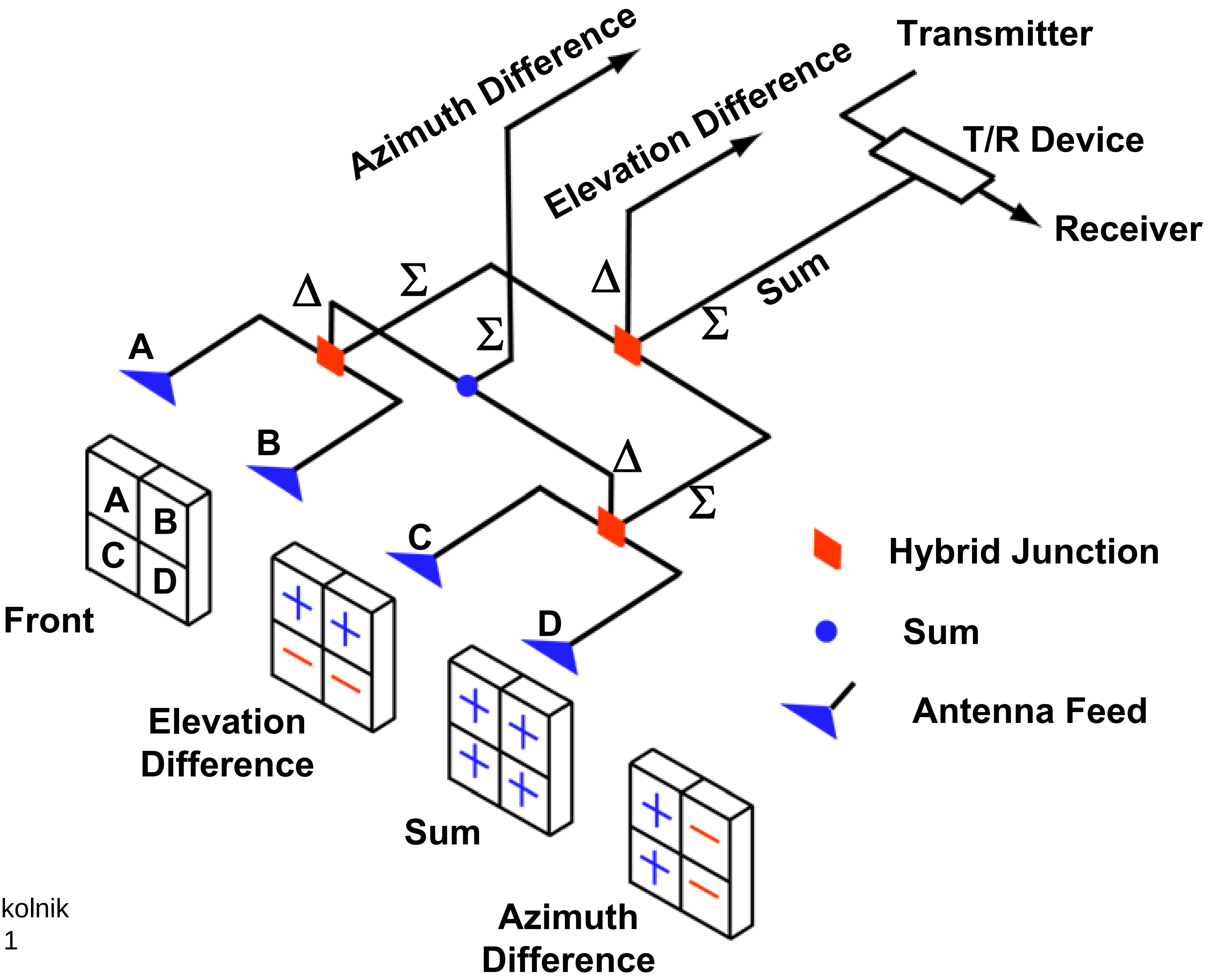
$$\text{Error Signal} = \frac{|\Delta|}{|\Sigma|}\cos\left(\phi_{\Sigma} - \phi_{\Delta}\right)$$



Error Signal vs. Angle

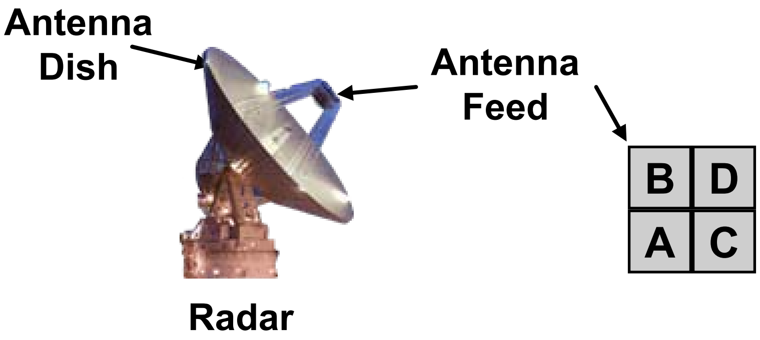
Adapted from Skolnik
Reference 1

Four Horn Monopulse Block Diagram

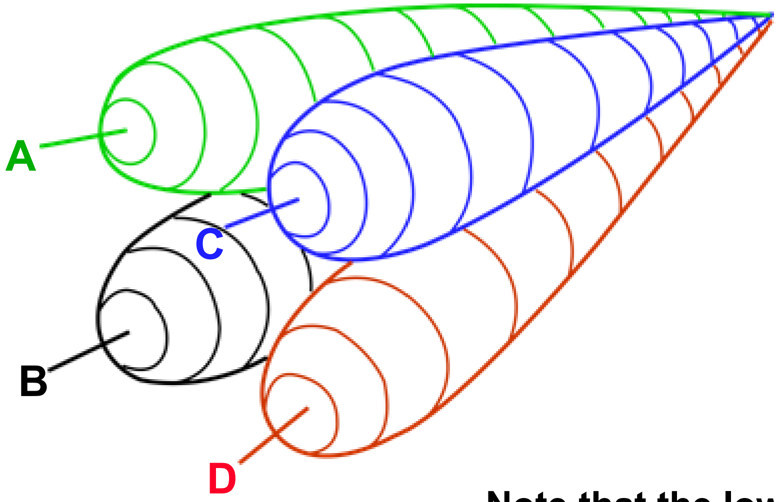


Adapted from Skolnik
Reference 1

Two Dimensional- Four Horn Monopulse



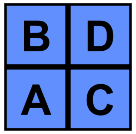
- Σ = Sum channel signal
  Δ = Difference channel signal
  ϕ = phase difference between Σ and Δ
- Error signal $e = \dfrac{|\Delta| \cos \phi}{|\Sigma|}$



Note that the lower feeds generate the upper beams

Sum beam

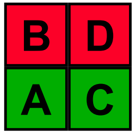
Σ



A+B+C+D

Elevation difference beam

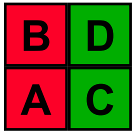
Δ_{EL}



B+D – (A+C)

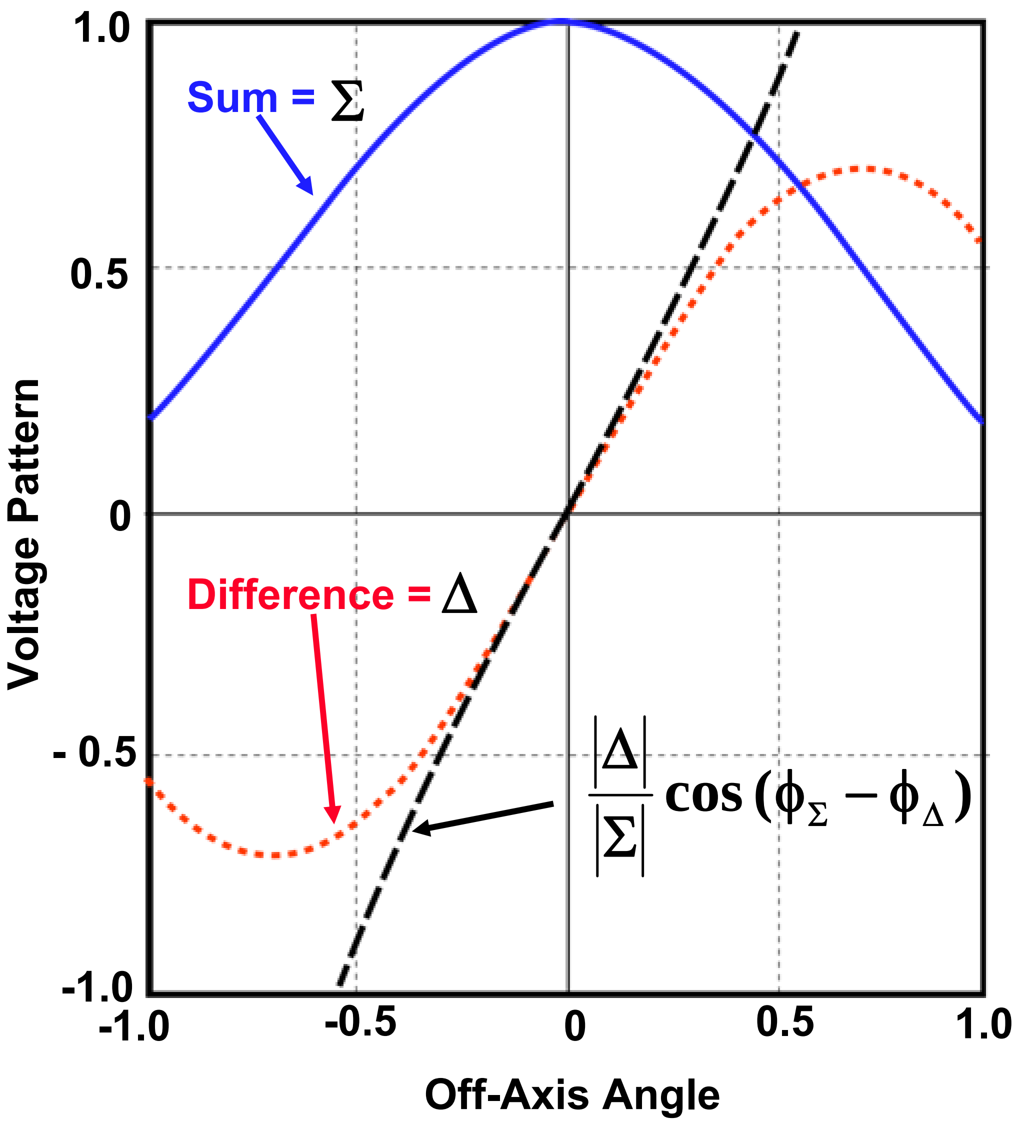
Azimuth difference beam

Δ_{AZ}

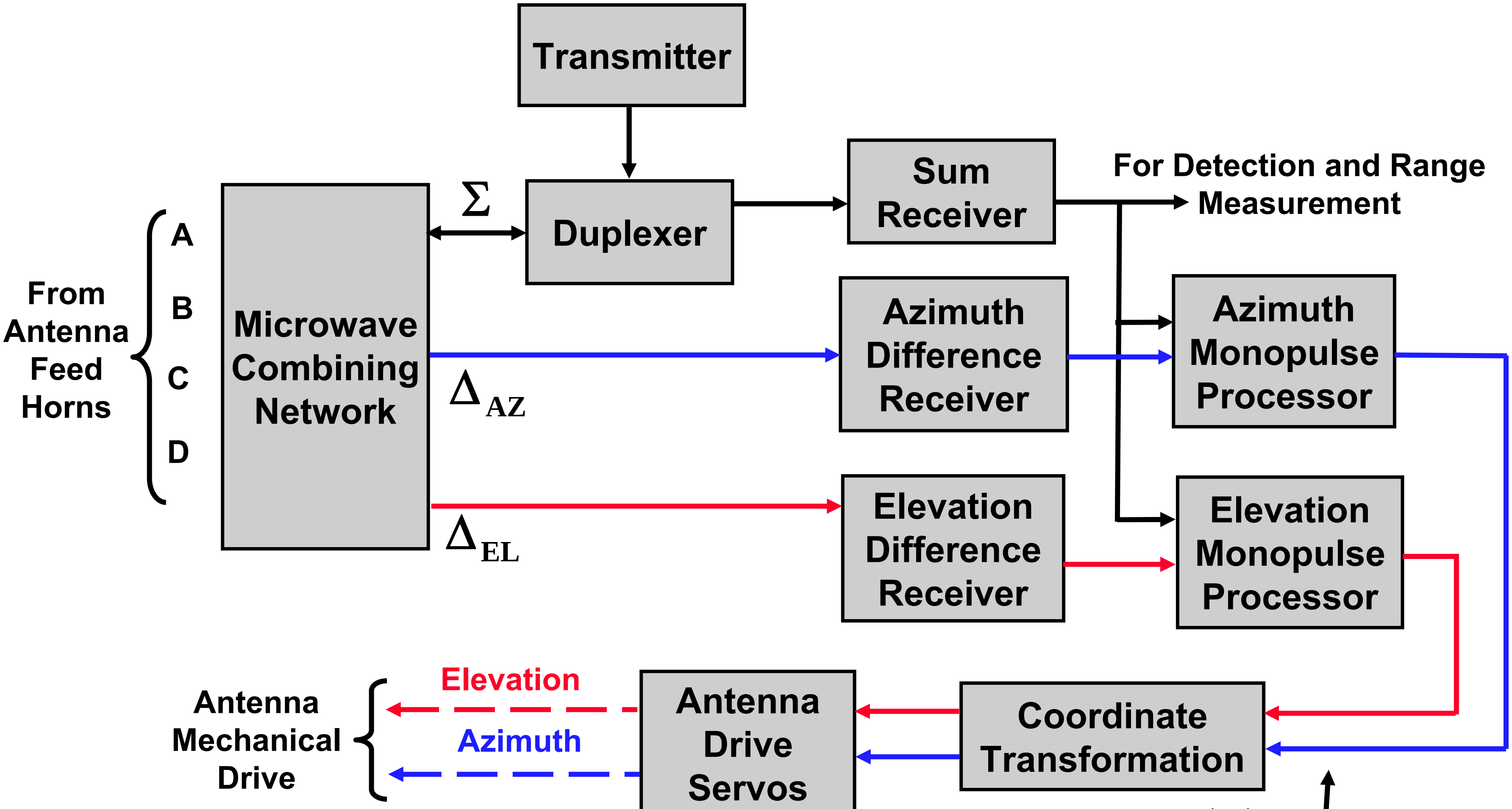


B+A – (C+D)

Monopulse Error Pattern



Functional Diagram of Monopulse Radar



Transmitter

Duplexer

Sum Receiver

For Detection and Range Measurement

From Antenna Feed Horns

A

B

C

D

Microwave Combining Network

Σ

Δ_{AZ}

Δ_{EL}

Azimuth Difference Receiver

Azimuth Monopulse Processor

Elevation Difference Receiver

Elevation Monopulse Processor

Coordinate Transformation

Antenna Drive Servos

Elevation

Azimuth

Antenna Mechanical Drive

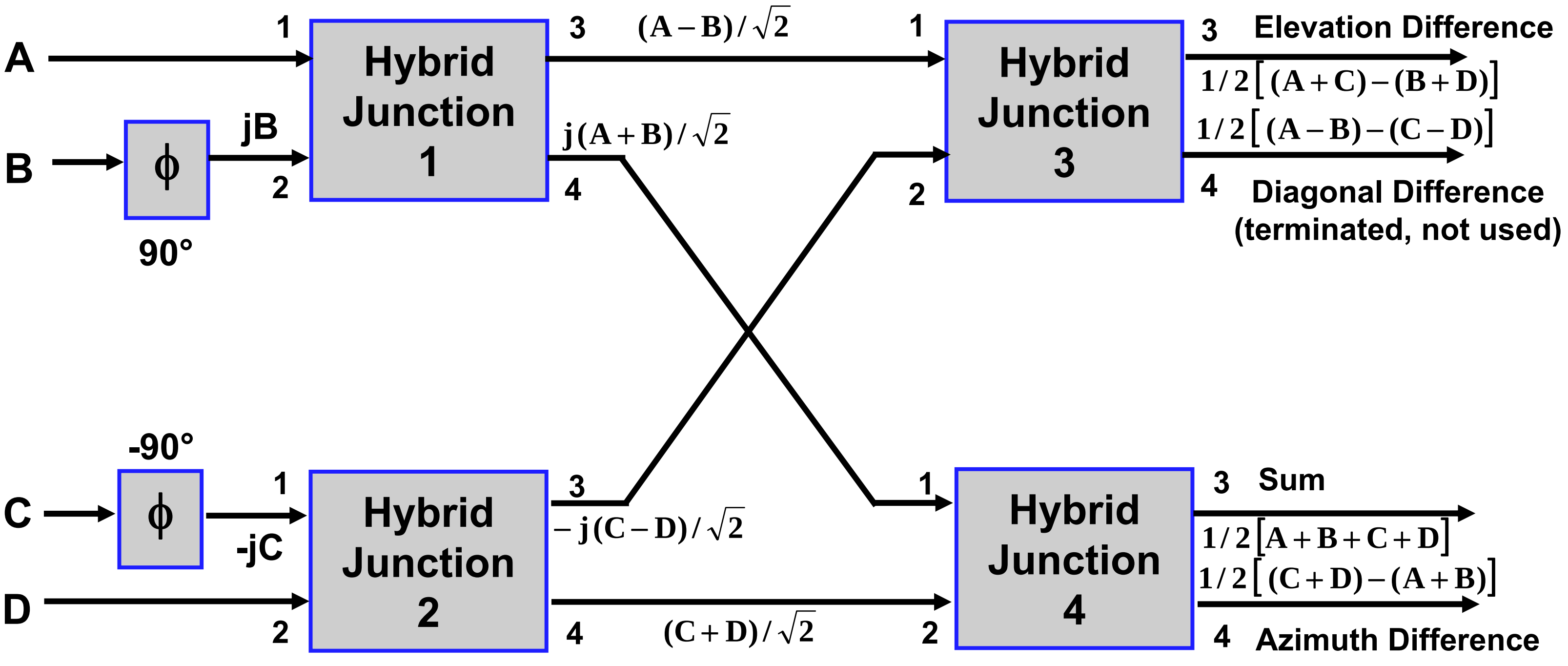
$$\mathrm{Error(az\;\&\;el)} = \frac{|\Delta|}{|\Sigma|}\cos\left(\phi_{\Sigma} - \phi_{\Delta}\right)$$

Adapted from Sherman
Reference 5

Microwave Combining Network
(Four Horn Monopulse Feed)

B	D
A	C

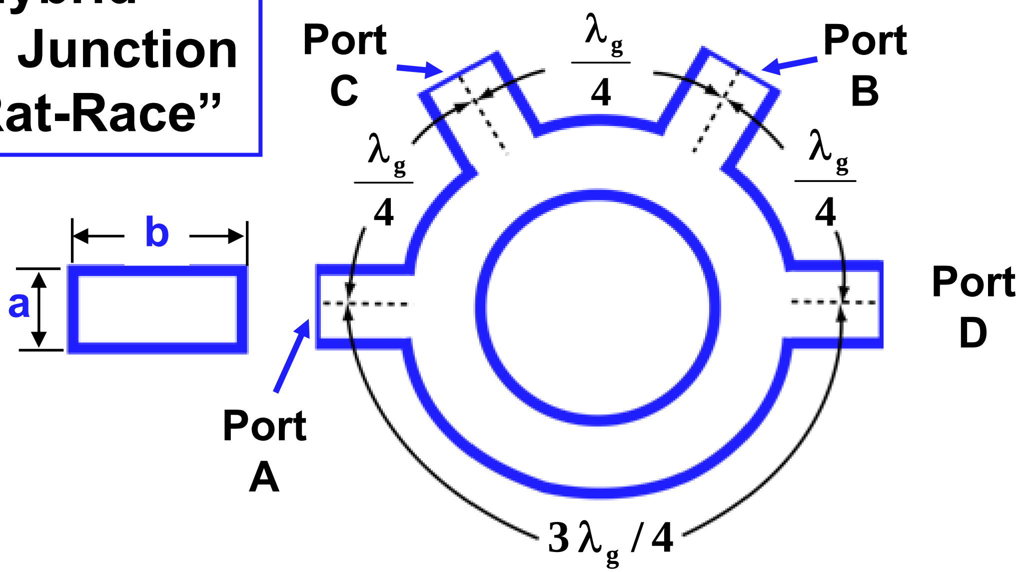
Arrangement Of Horns



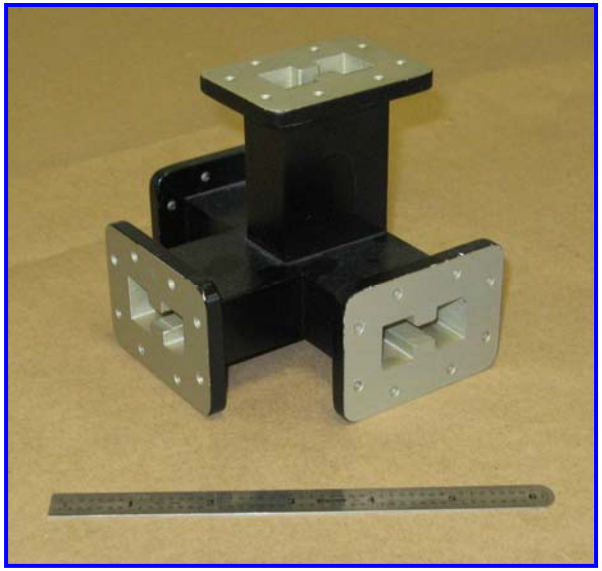
Adapted from Sherman
Reference 5

Three Types of Hybrid Junctions

Hybrid Ring Junction or "Rat-Race"



Magic - T



Courtesy of Cobham Sensor Systems.
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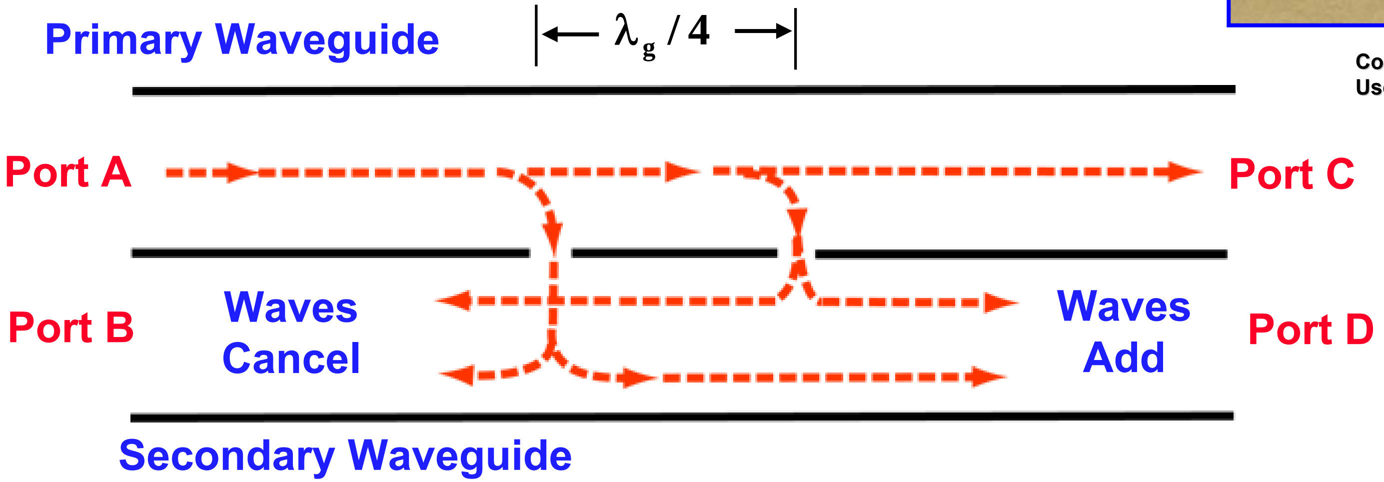
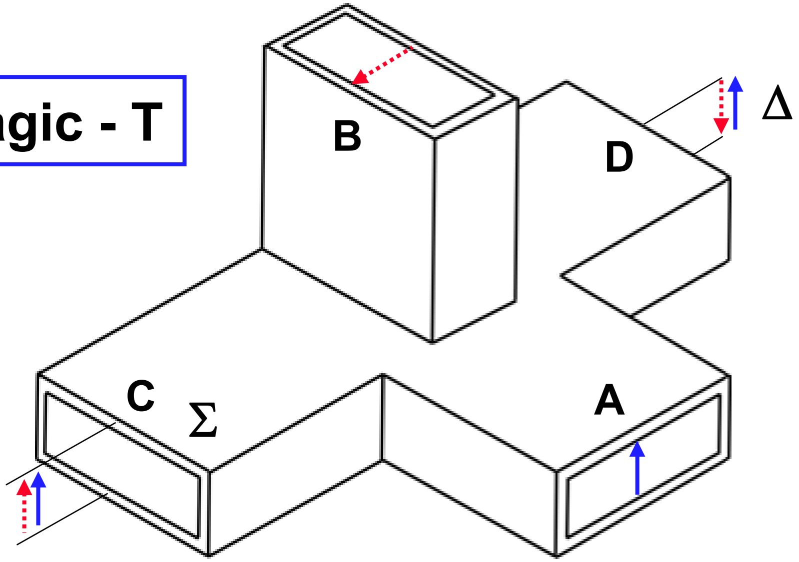


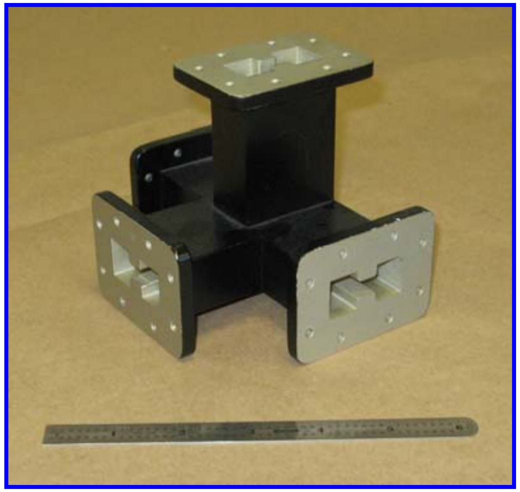
3 dB Directional Coupler

Hybrid Junctions for Monopulse Radars

Magic - T



Photograph of
C - Band Magic - T
(Ridged waveguide)



Courtesy of Cobham Sensor Systems.
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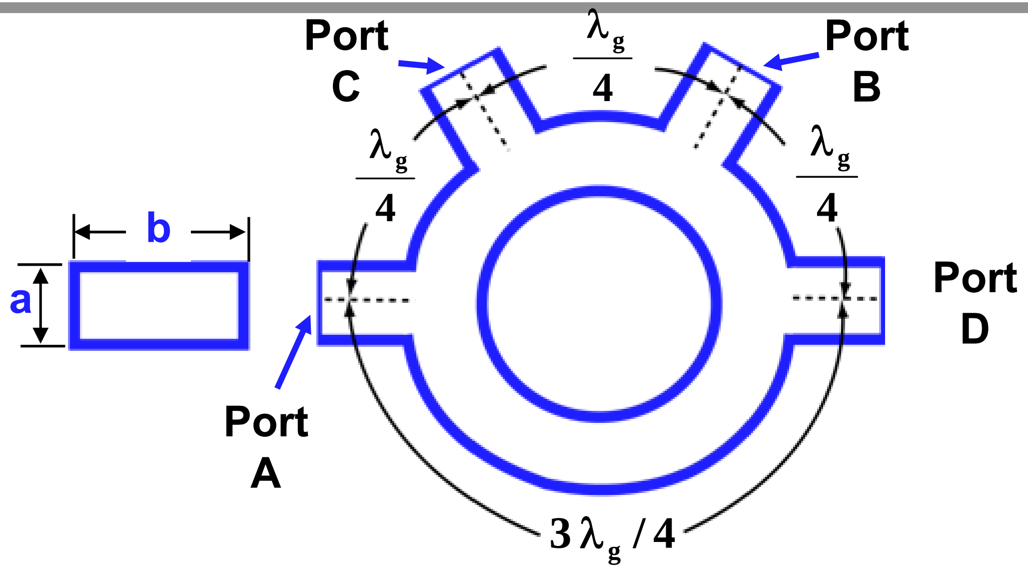
- A signal input at port A divides equally in amplitude and phase between ports C and D, but does not appear at port B
  - Port B cannot support that propagation mode
- A signal input to port B divides equally but with opposite phases between ports C and D
  - Does not appear at port A
- If inputs are applied simultaneously to ports A and B, their sum will appear at port C and the difference at the D

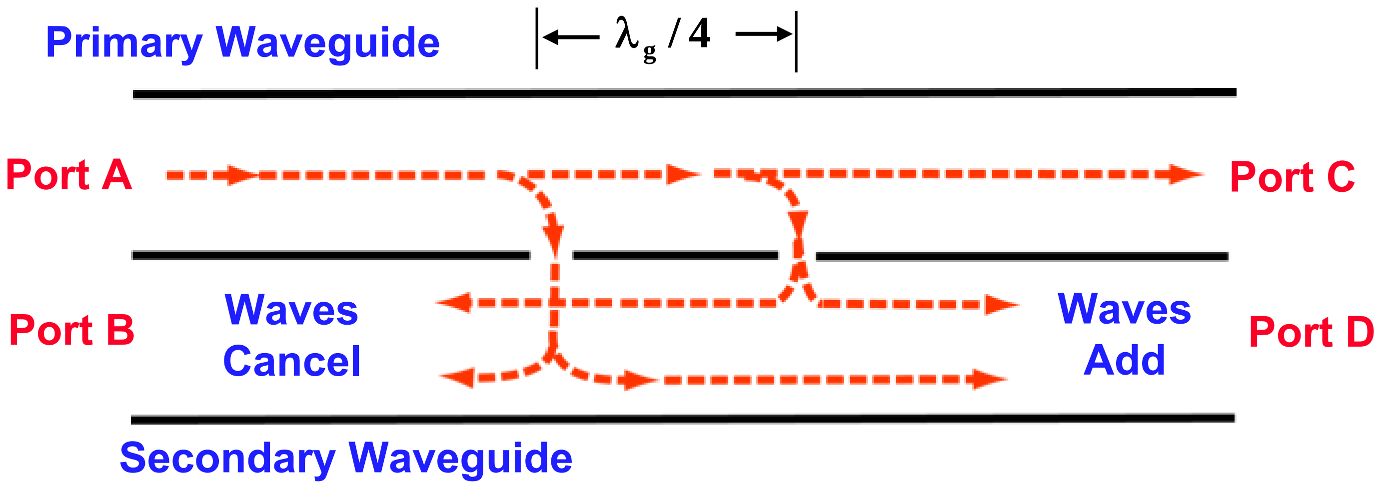
Hybrid Junctions Used in Monopulse Radar



Hybrid Ring Junction or "Rat-Race"

- A signal input at port A reaches output port D by two separate paths, which have the same path length ($3\lambda/4$)
  - The two paths reinforce at port D
- An input signal at port B reaches output port D through paths differing by one wavelength ($5\lambda/4$ and $\lambda/4$)
  - The two paths reinforce at port D
- Paths from A to D and B to D differ by 1/2 wavelength
  - Signal at port A - signal at port B will appear at port D
- If signals of the same phase are entered at A and , the outputs C and D are the sum (Σ) and difference (Δ).

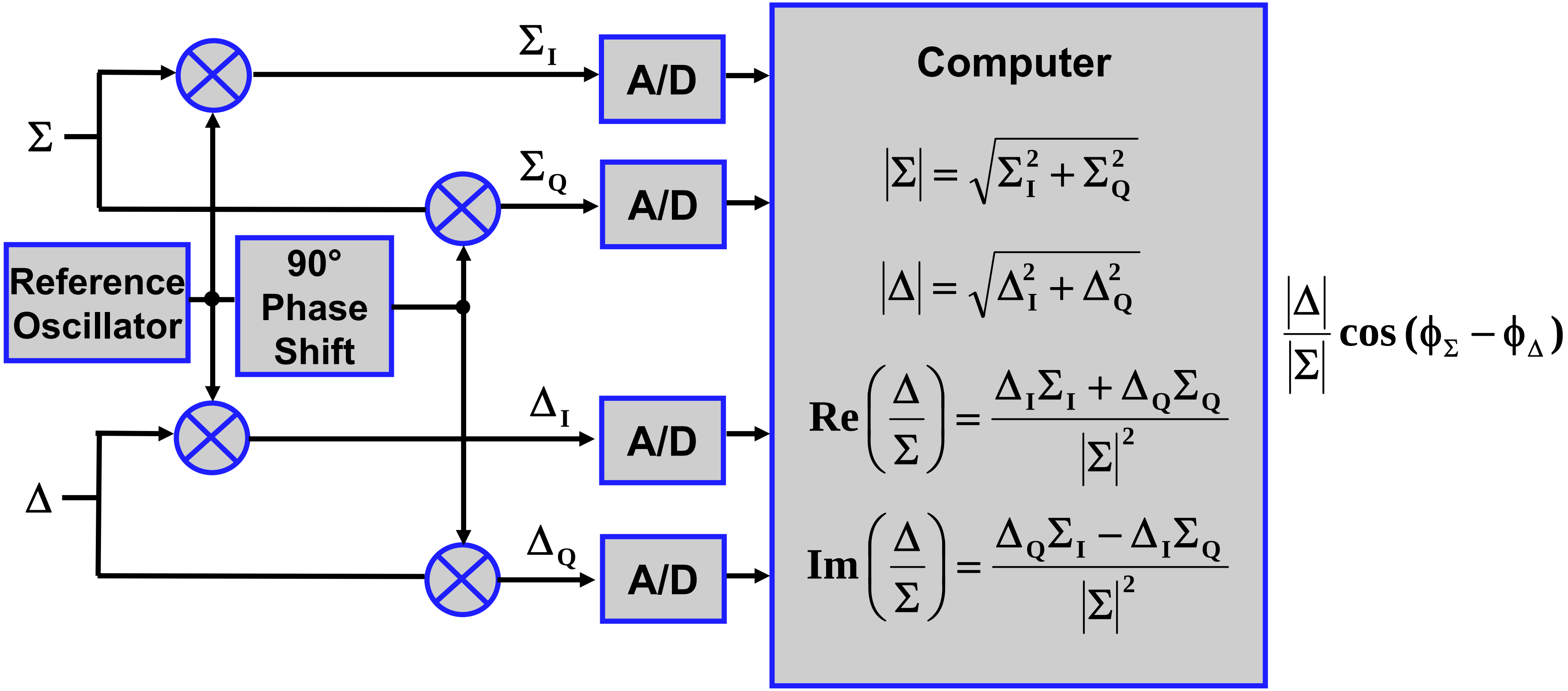
Hybrid Junctions Used in Monopulse Radar



3 dB Directional Coupler

- This coupler is made by aligning two rectangular waveguides with their walls touching
- Microwave energy from one of the waveguides is coupled to the other by means of appropriate holes or slots between the two waveguides
  - Because of the quarter wave spacing between the two slots, this configuration is frequency sensitive
  - A 90 degree phase shift has to be inserted in either port A or B in order to provide the sum and difference at ports C and D

Monopulse Processor



$$|\Sigma| = \sqrt{\Sigma_I^2 + \Sigma_Q^2}$$

$$|\Delta| = \sqrt{\Delta_I^2 + \Delta_Q^2}$$

$$\mathrm{Re}\left(\frac{\Delta}{\Sigma}\right) = \frac{\Delta_I \Sigma_I + \Delta_Q \Sigma_Q}{|\Sigma|^2}$$

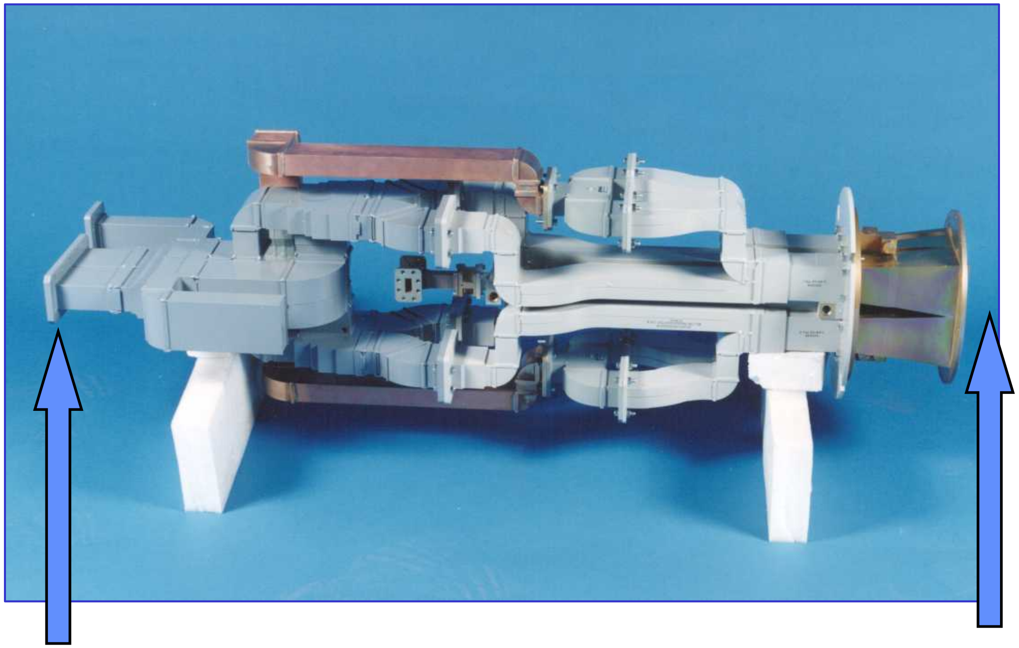
$$\mathrm{Im}\left(\frac{\Delta}{\Sigma}\right) = \frac{\Delta_Q \Sigma_I - \Delta_I \Sigma_Q}{|\Sigma|^2}$$

$$\frac{|\Delta|}{|\Sigma|} \cos\left(\phi_\Sigma - \phi_\Delta\right)$$

Adapted from Sherman
Reference 5

S Band Monopulse Feed with X Band Center Feed

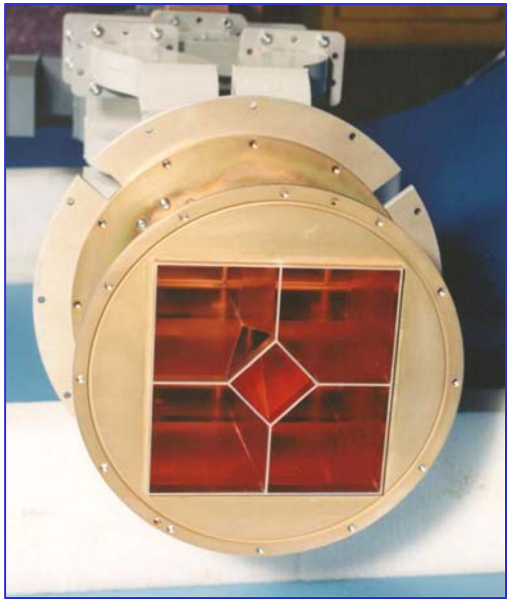
Side View



From S and X Band Transmitters

Output

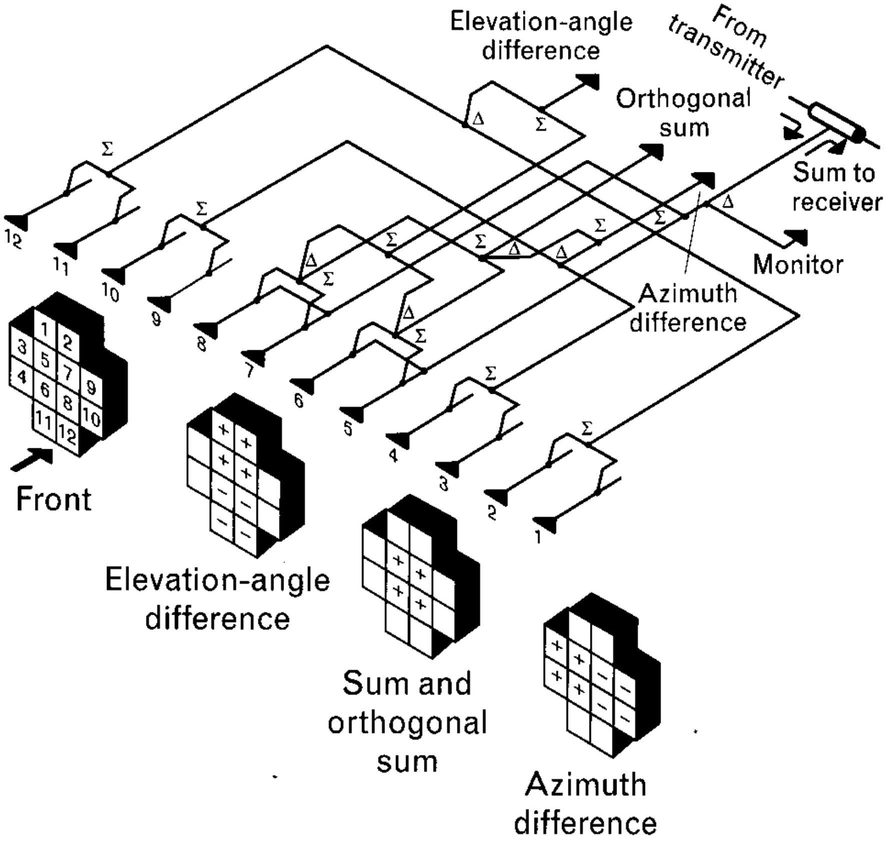
Four Horn Monopulse S band Feed (X band Feed at center)

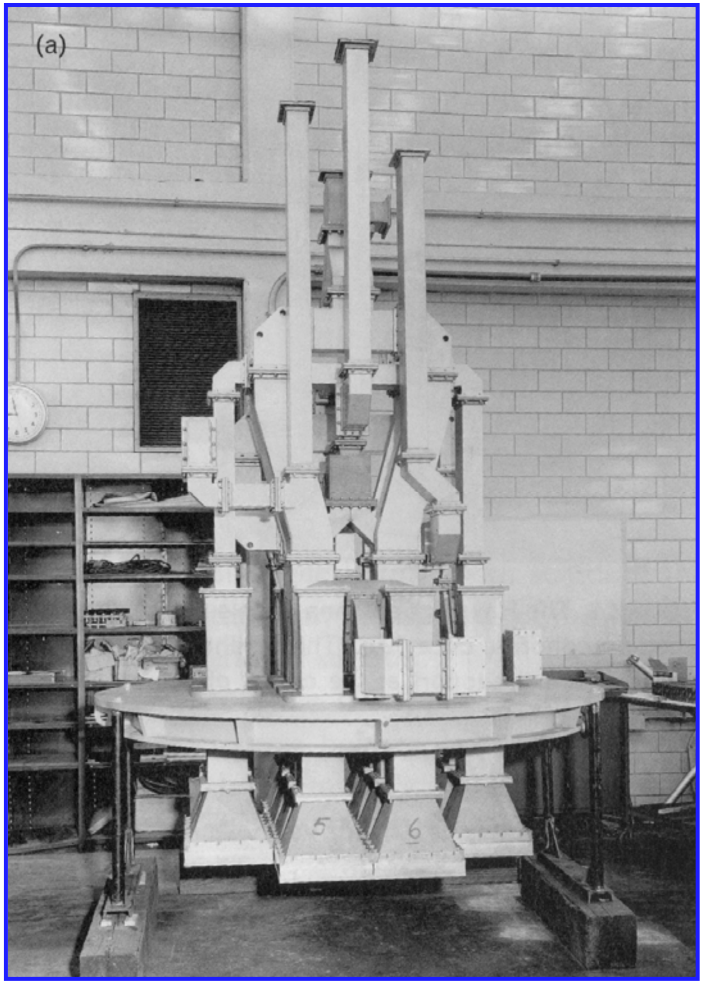


Front View of Output

Courtesy of MIT Lincoln Laboratory, Used with Permission

Twelve Horn Monopulse Feed





Photograph of 12 Horn Monopulse Feed

Courtesy of MIT Lincoln Laboratory, Used with Permission

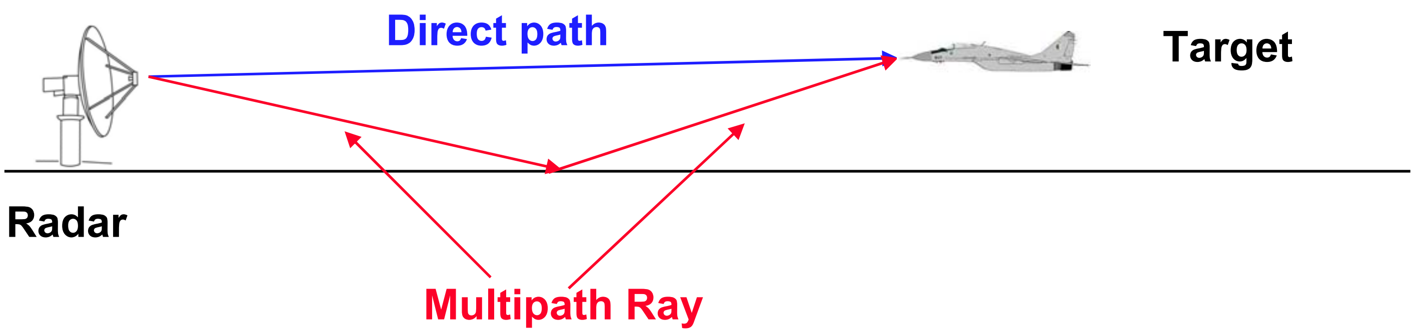
Glint (Angle Noise)

- Glint, or angle noise, is a fluctuation or error in the angle measurement caused by the radar’s energy reflecting from a complex target with multiple scattering centers
  - It causes a distortion of the echo wavefront
  - The result of having a non-uniform wavefront from a complex target, when the radar was designed to process a planar echo wavefront, is an error in the measurement of the angle of arrival
  - The measured angle of arrival can often cause the boresight of the tracking antenna to point outside the angular extent of the target, which can cause the radar to break track

- Glint can be a major source of error when making angle measurements
  - Short range where angular extent of target is large

- Problem for all tracking radars with closed loop angle tracking
  - Monopulse, conical scan, sequential lobing

Low Angle Tracking



- **The target is illuminated via two paths (direct and reflection)**

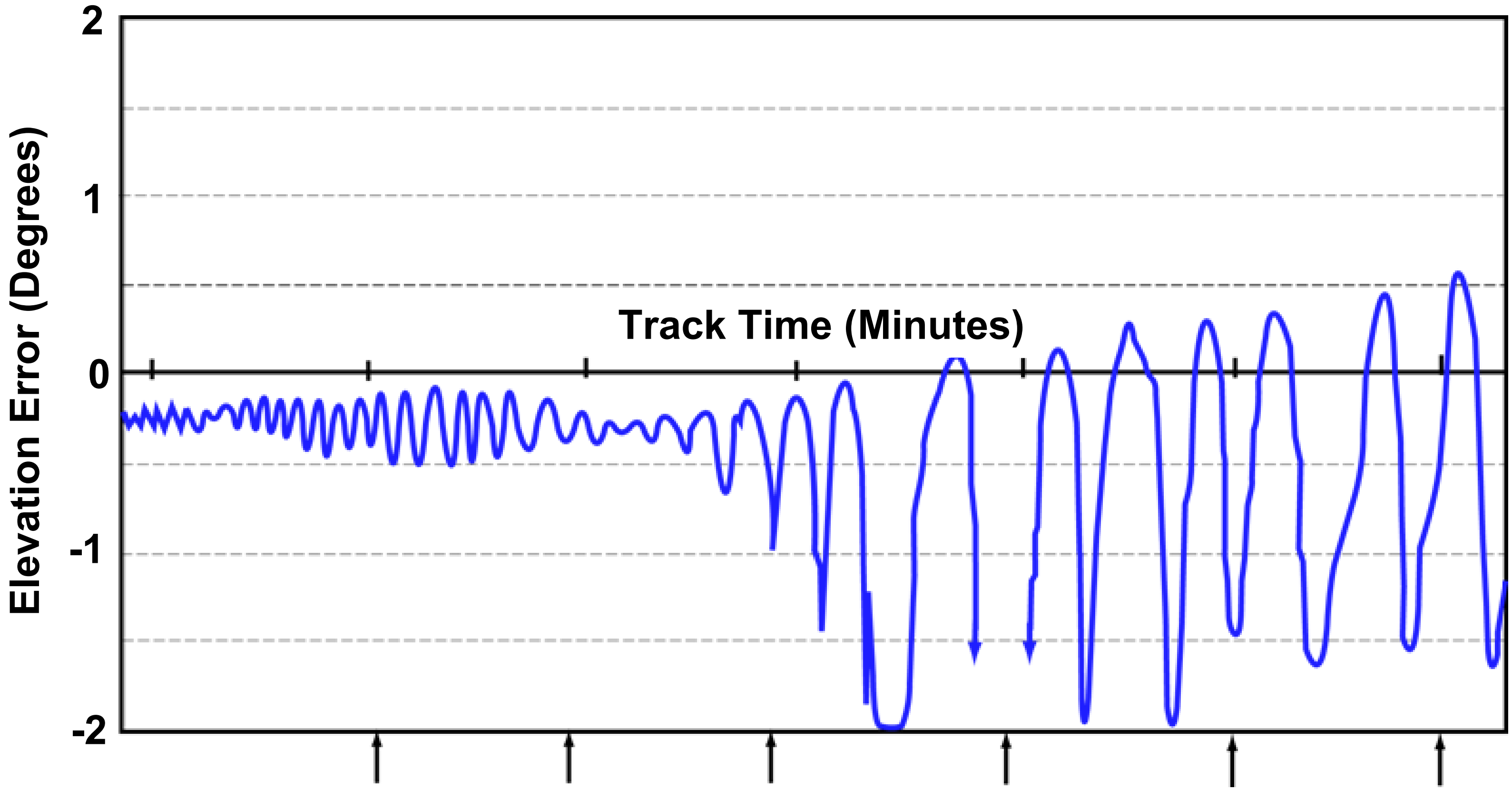
- **Error in measured elevation angle occurs because of glint**
  - **At low grazing angles, reflection coefficient close to -1**

- **Tracking of targets at low elevation angles can produce significant errors in the elevation angle and can cause loss of track**

- **The surface reflected signal is sometimes called the multi-path signal and the glint error due to this geometry a multi-path error**

Measured Low Angle Tracking Error

Aircraft Tracked by S-Band Phased Array radar (FPS-16 provided “Truth)



Azimuth (deg)	32.5	30.7	30.4	30.3	30.3	30.2
Elevation (deg)	3.0	2.4	2.0	1.7	1.4	1.3
Range (km)	19.9	24.3	28.9	33.5	38.1	42.7

Adapted from Skolnik
Reference 1

Outline

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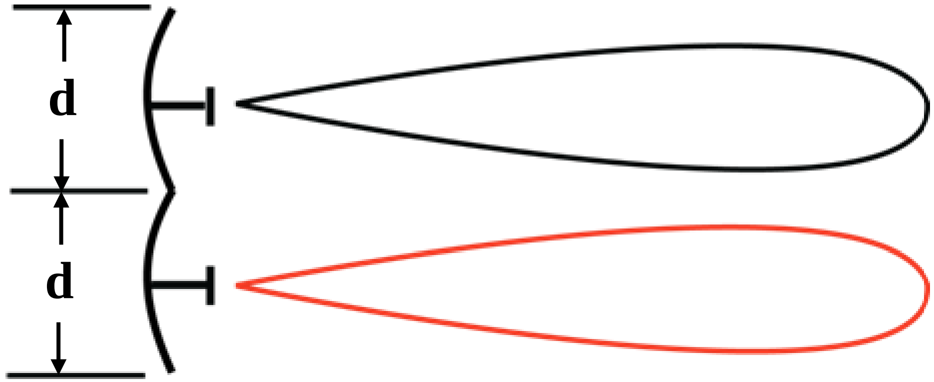
- Observable Estimation
- Single Target Tracking
  - Angle tracking techniques
    - Amplitude monopulse
    - ➡ Phase comparison monopulse
    - Sequential lobing
    - Conical scanning
  - Range tracking
  - Servo systems
- Multiple Target Tracking
- Summary

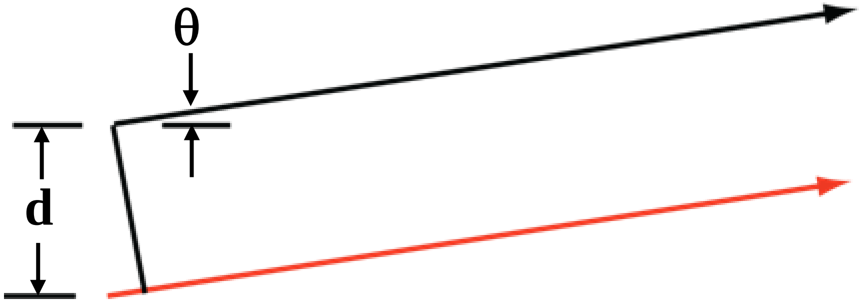
Phase Comparison Monopulse



Two antennas radiating identical beams in the same direction

Also known as "interferometer radar"

Geometry of the signals at the two antennas when received from a target at an angle θ

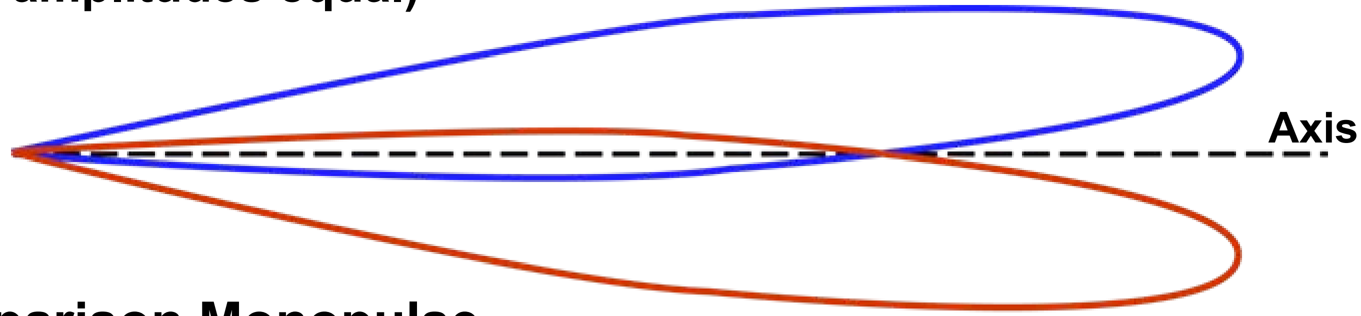


The phase difference of the signals received from the two antennas is :

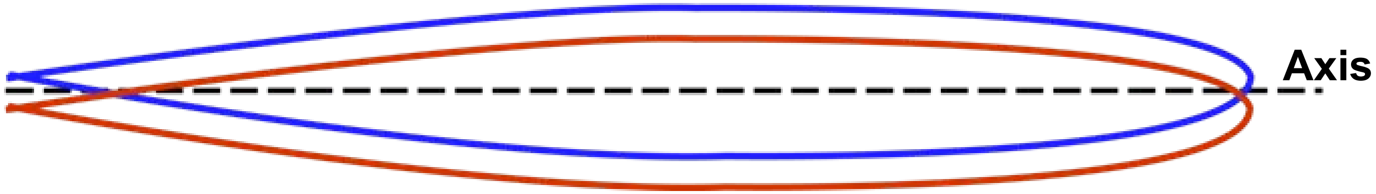
$$\Delta\phi = 2\pi \frac{d}{\lambda} \sin\theta$$

Comparison of Monopulse Antenna Beams

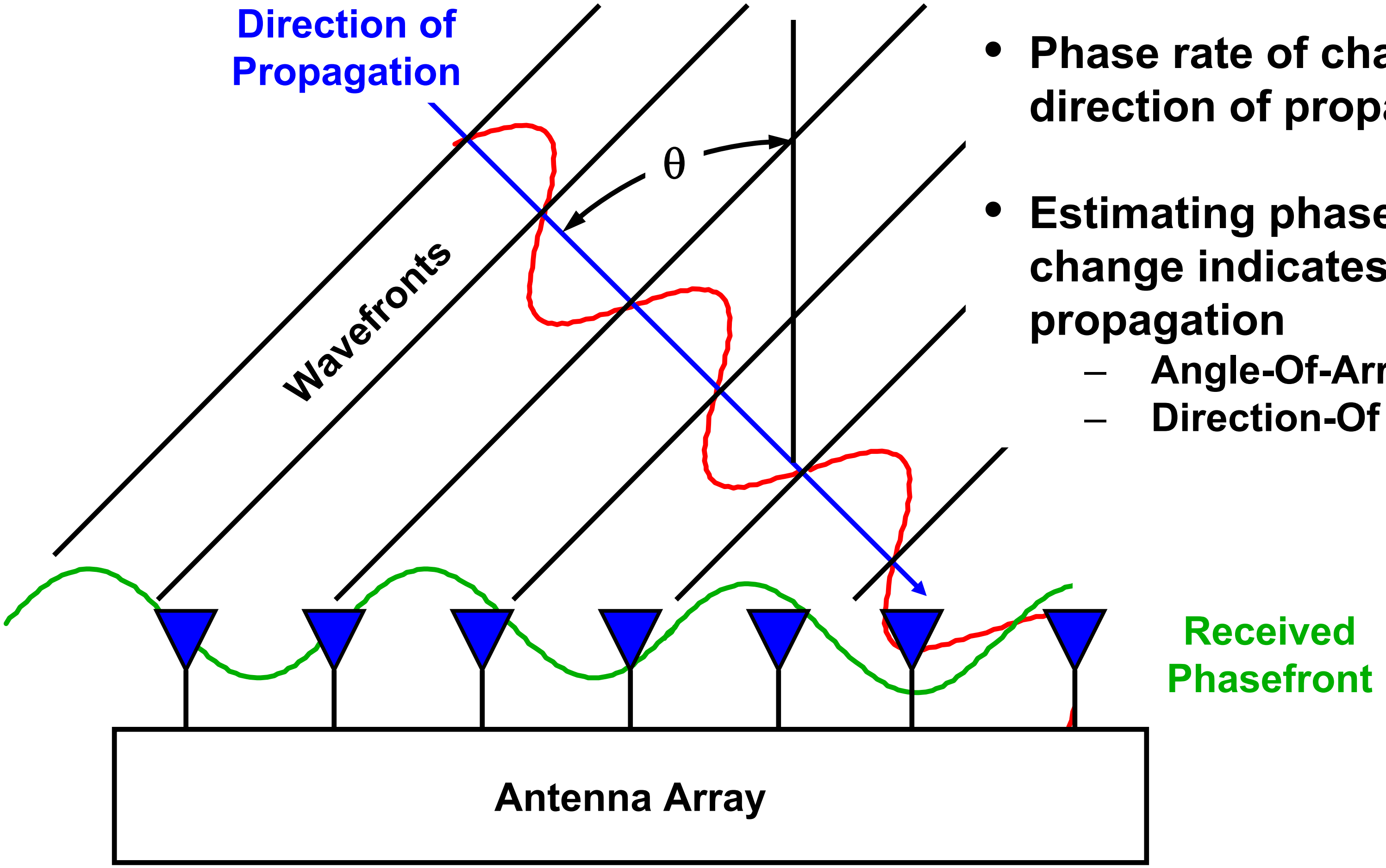
- Amplitude Comparison Monopulse
  - Common phase center, beams squinted away from axis
  - Target produces signal with same phase but different amplitudes (On axis amplitudes equal)



- Phase Comparison Monopulse
  - Beams parallel and identical
  - Lateral displacement of phase center much greater than λ
  - Target produces signal with same amplitude but different phase (On axis phases equal)
  - Grating lobes and high sidelobes a problem



Angle Estimation with Antenna Arrays



- **Received signal varies in phase across array**
- **Phase rate of change related to direction of propagation**
- **Estimating phase rate of change indicates direction of propagation**
  - **Angle-Of-Arrival (AOA)**
  - **Direction-Of Arrival (DOA)**

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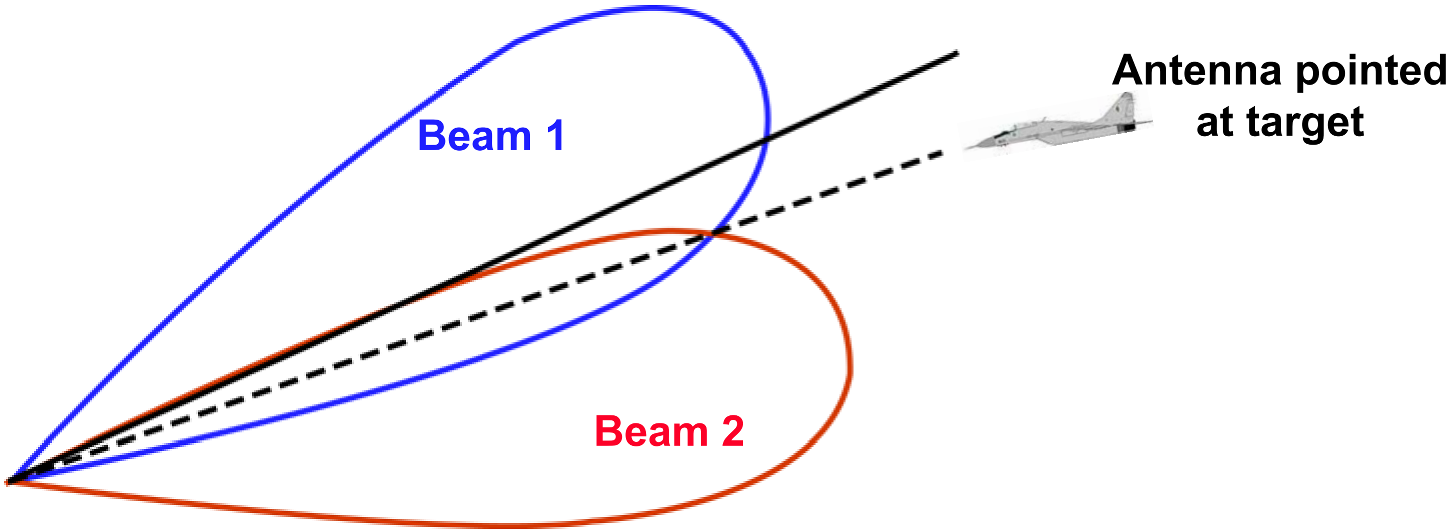
Sequential Lobing Angle Measurement



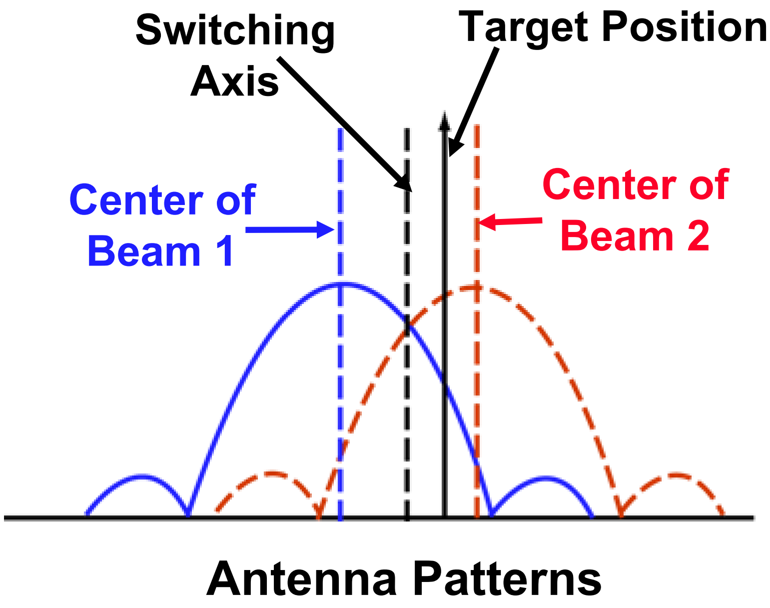
V_1 = voltage from upper beam (lobe)
V_2 = voltage from lower beam (lobe)

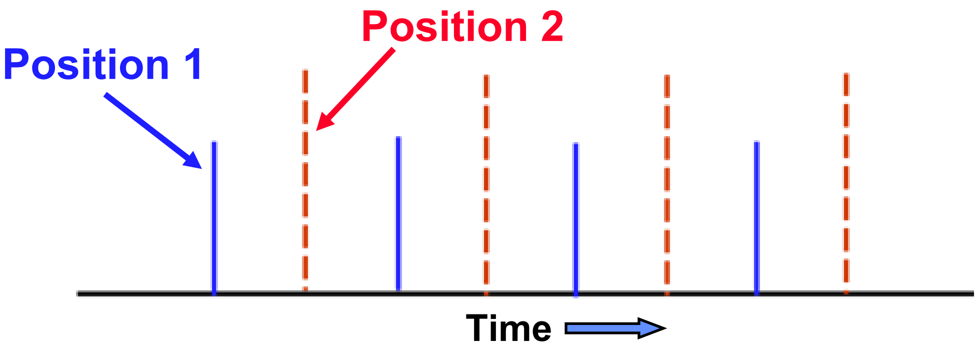
If $V_1 - V_2 > 0$ Antenna pointing to high
If $V_1 - V_2 < 0$ Antenna pointing to low
If $V_1 - V_2 = 0$ Antenna pointed at target

- **The Sequential Lobing angle tracking technique time shares a single antenna beam to obtain the angle measurement in a sequential manner**

Adapted from Sherman
Reference 5

Sequential Lobing Angle Measurement





- The differences in echo signals between the two switched beams is a measure of the angular displacement of the target from the switching axis
  - The beam with the larger signal is closer to the target
  - A control loop is used to redirect the beam track locations to equalize the beam response
  - When the echo signals in the two beam positions are equal, the target is on axis

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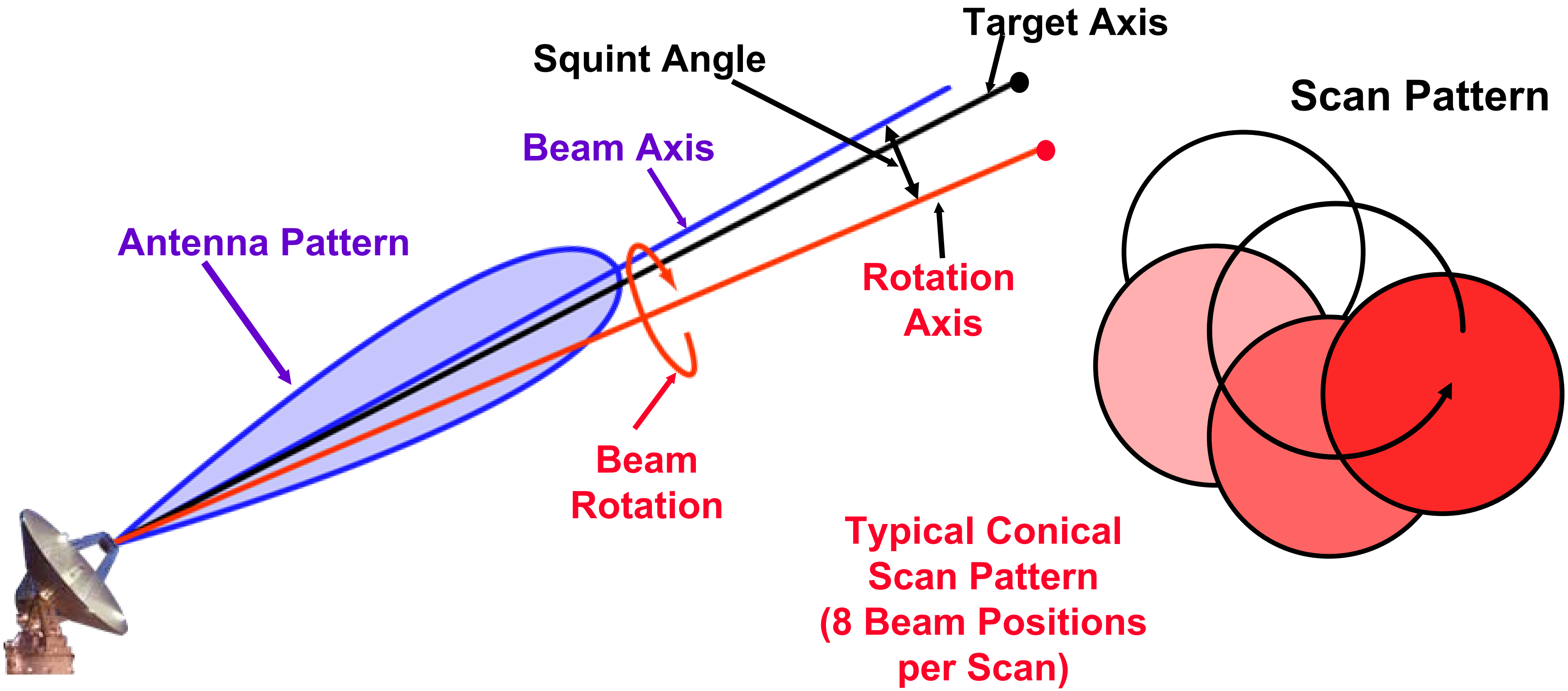
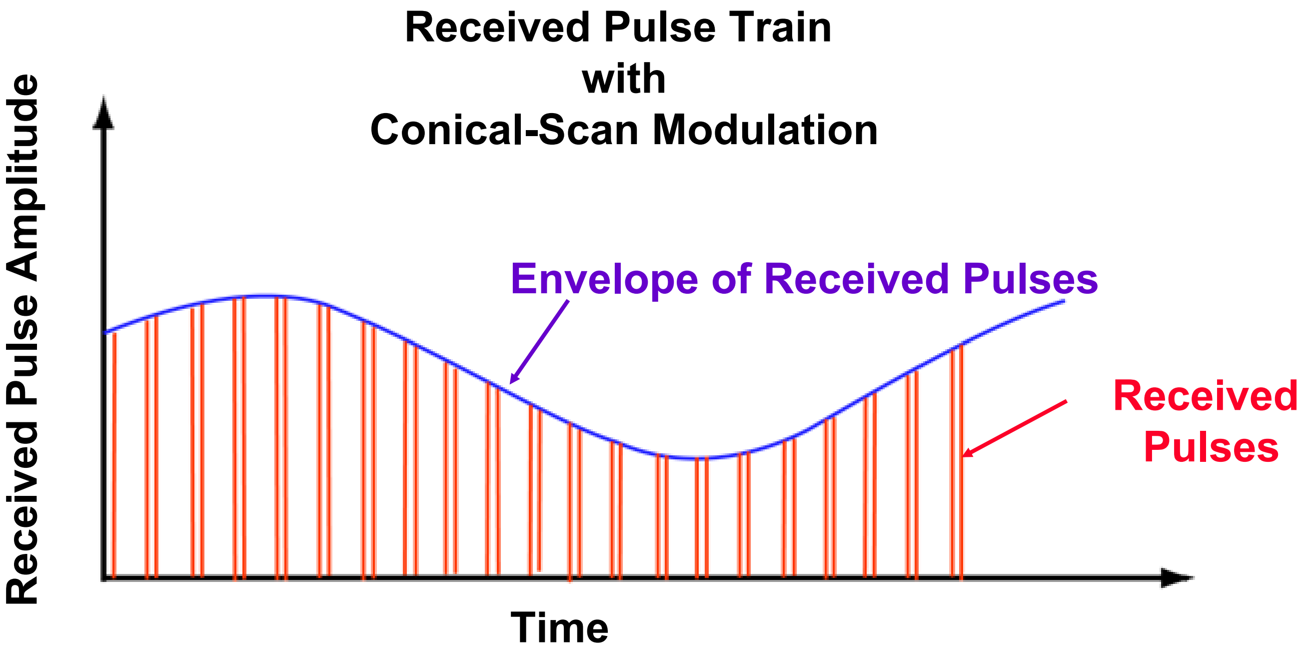
Conical Scan Tracking Concept



- The angle between the axis of rotation and the axis of the antenna beam is the squint angle
- Because of the rotation of the squinted beam and the targets offset from the rotation axis, the amplitude of the echo signal will be modulated at a frequency equal to the beam rotation

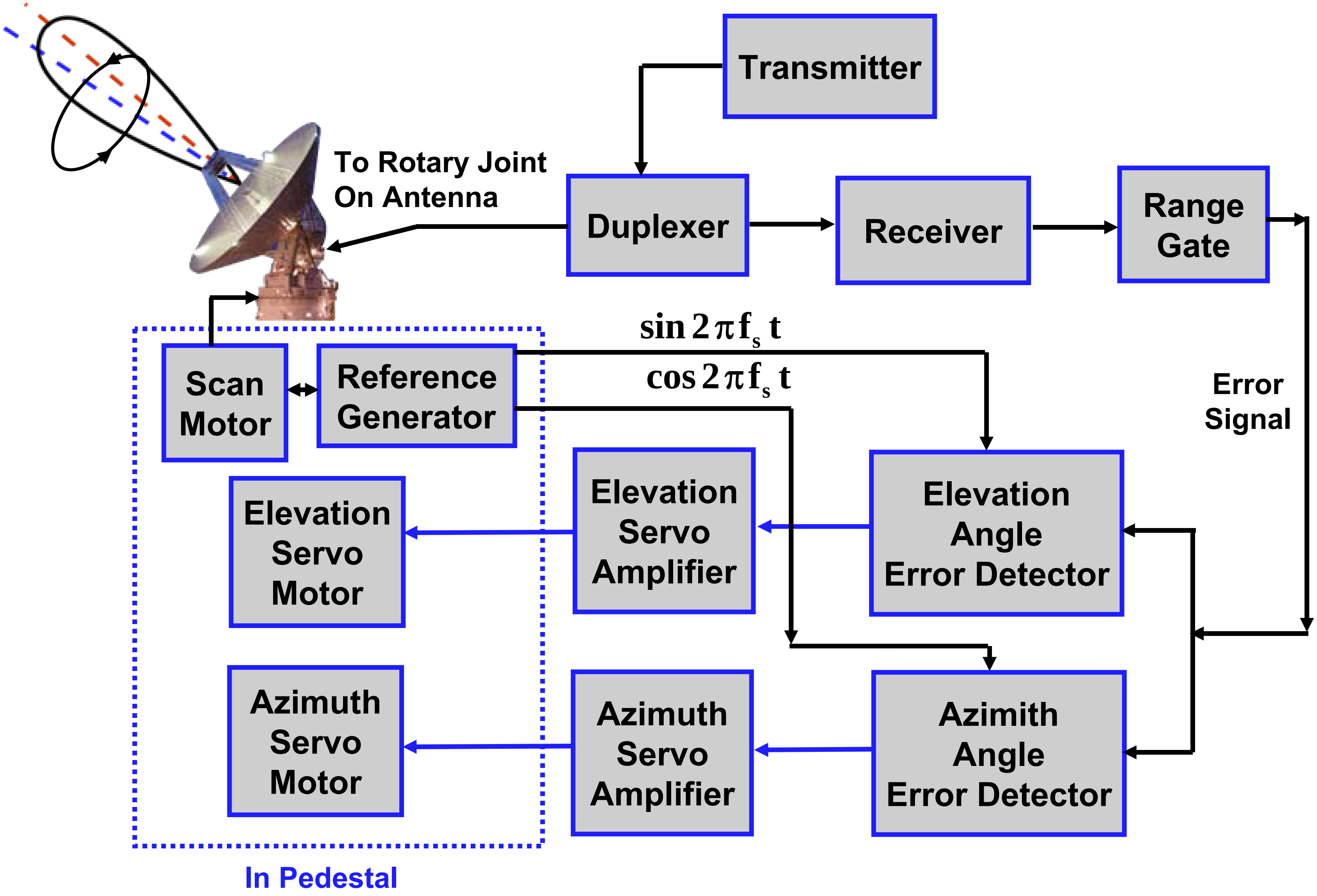
Conical Scan Pulse Trains



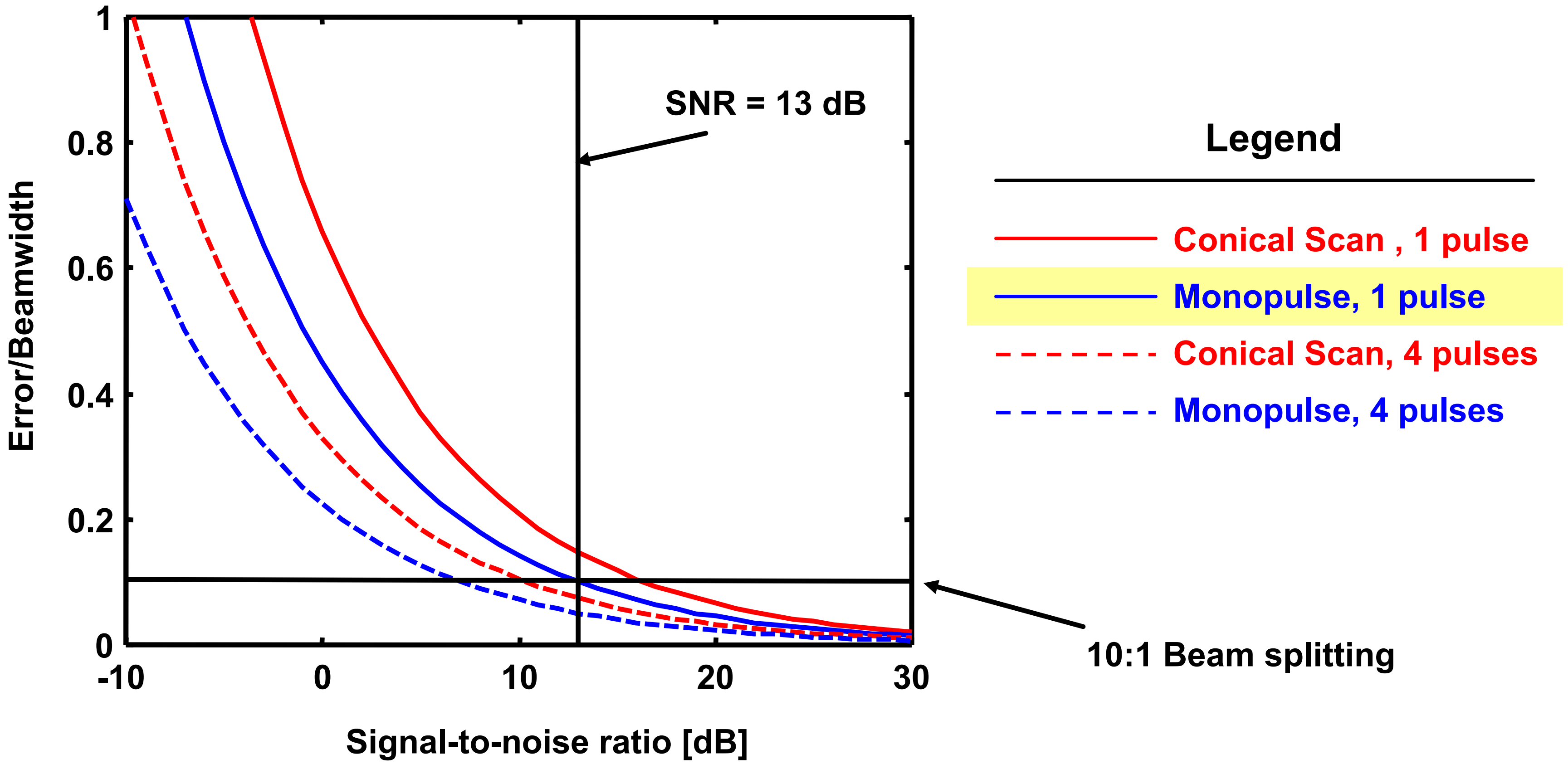
- The amplitude of the modulation is proportional to the angular distance between the target direction and the rotation axis
  - Beam displacement

- The phase of the modulation relative to the beam scanning position contains the direction information
  - Angle error

Block Diagram of Conical Scan Radar



Beam-Splitting



At typical detection threshold levels (~13 dB) the resolution cell can be approximately split by a factor of ten; i.e. 10:1 antenna beam splitting

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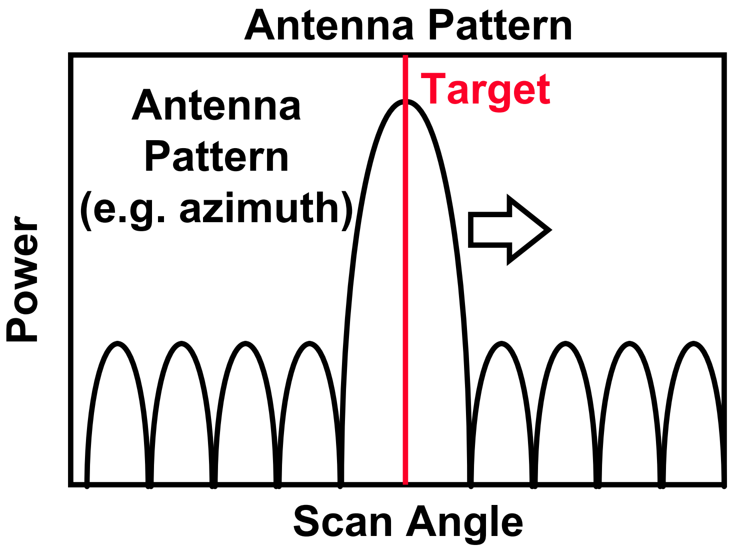
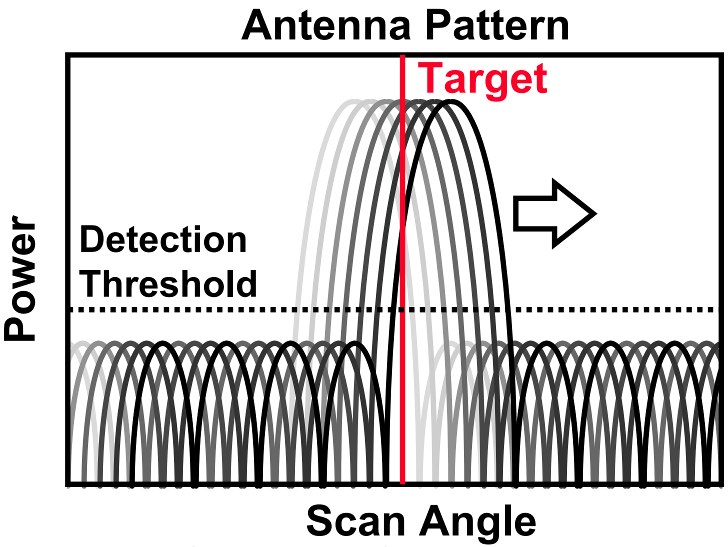
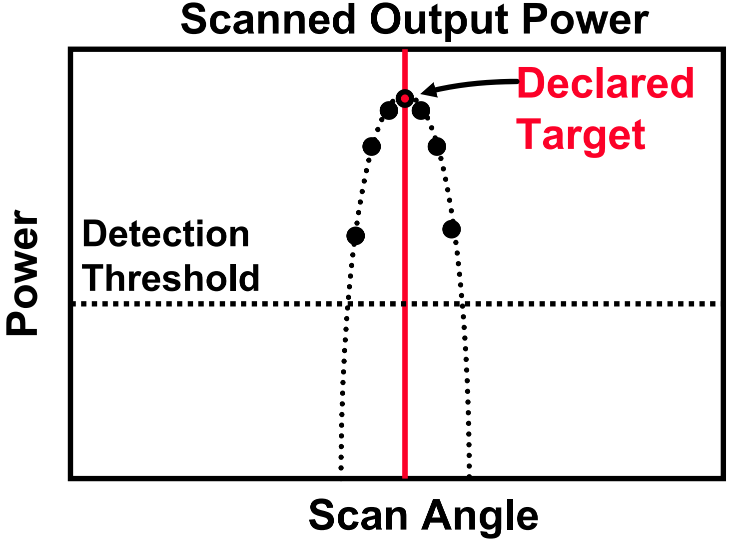
Angle Estimation with Scanning Radar
(Multiple Pulse Angle Estimation)





Courtesy US Dept of Commerce

Airport Surveillance Radar



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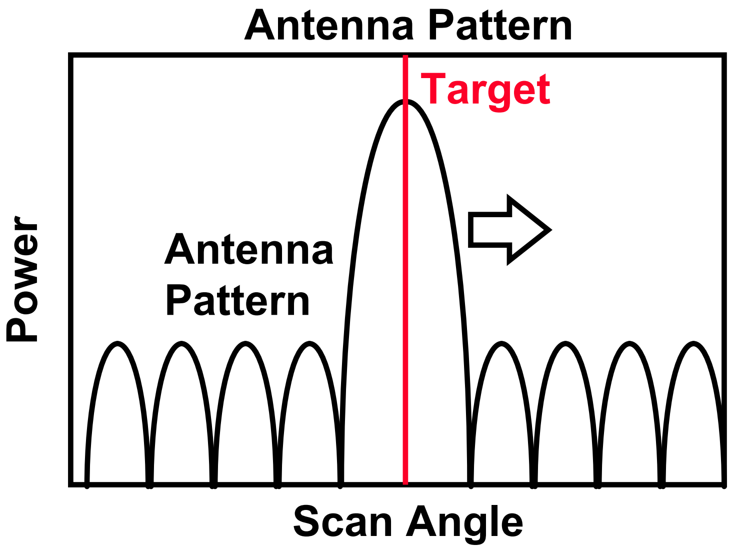
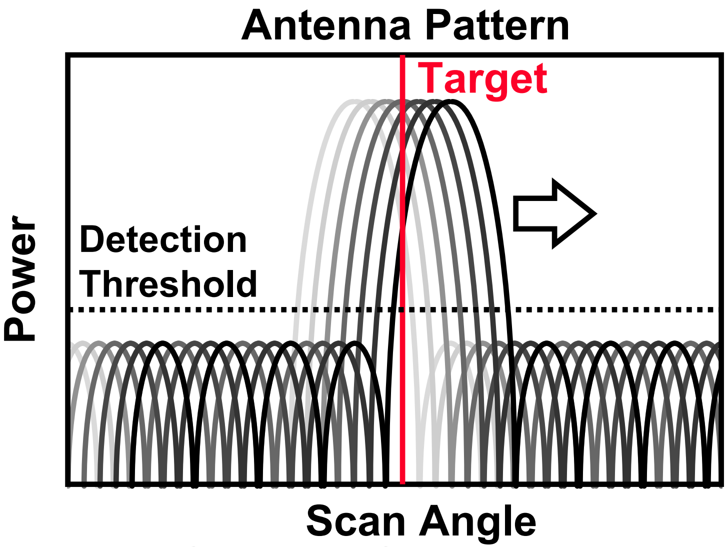
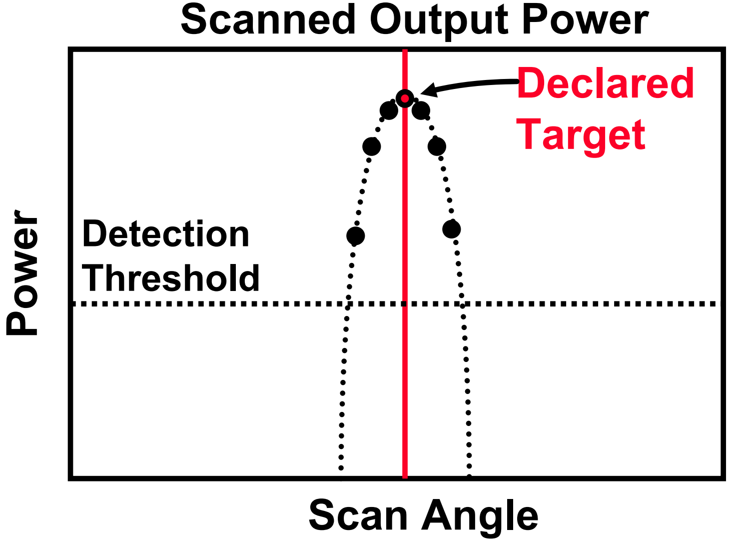
Angle Estimation with Scanning Radar
(Multiple Pulse Angle Estimation)



- **For a “track-while scan” radar, the target angle is measured by:**
  - **Fitting the return angle data from different angles to the known antenna pattern, or**
  - **Using the highest amplitude target return as the measured target angle location**





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Angle Estimation with Array Antennas

- Phased array radars are well suited for monopulse tracking
  - Amplitude Comparison Monopulse
    - Radiating elements can be combined in 3 ways
    - Sum, azimuth difference, and elevation difference patterns
  - Phase Comparison Monopulse
    - Use top and bottom half of array for elevation
    - Use right and left half of array for azimuth
- Lens arrays (e.g. MOTR) would use amplitude monopulse
  - Four-port feed horn would be same as for dish reflector

BMEWS

MOTR

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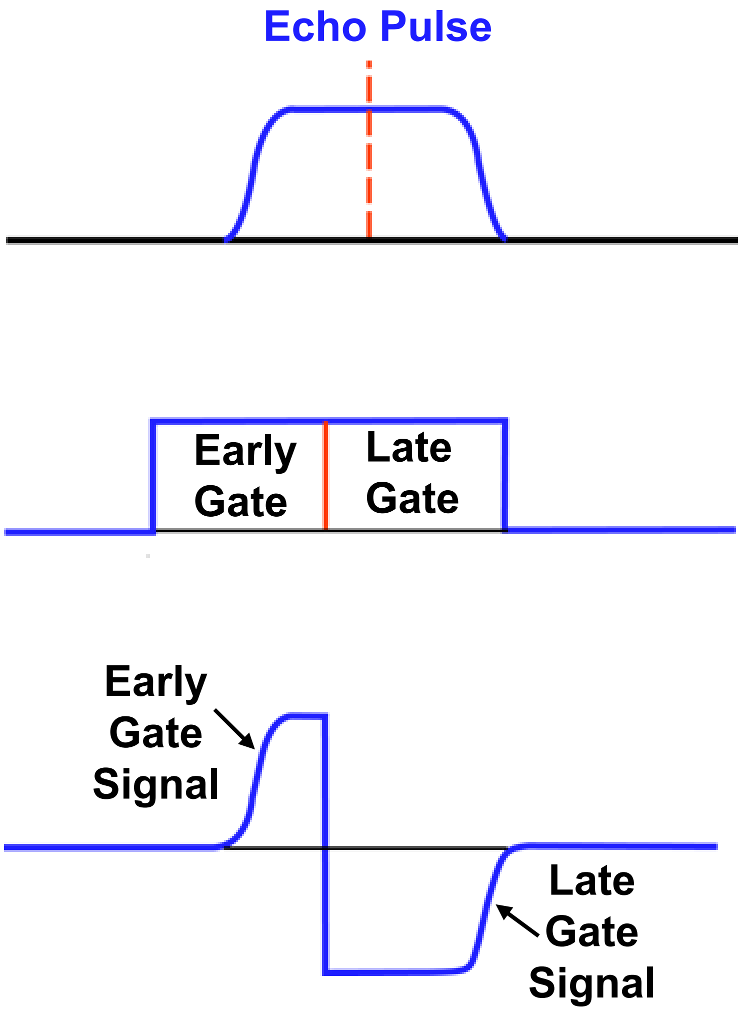
Split Gate Range Tracking



Difference Signal between Early and Late Range Gates

- Two gates are generated; one is an early gate, the other is a late gate.
- In this example, the portion of the signal in the early gate is less than that of the late gate.
- The signals in the two gates are integrated and subtracted to produce the difference error signal.
- The sign of the difference indicates the direction the two gates have to be moved in order to have the pair straddle the echo pulse
- The amplitude of the difference determines how far the pair of gates are from the centroid.

Multi Target Tracking in Range, Angle, and Doppler

- Single target angle trackers (Dish radars) can be configured to track other targets in the radar beam
  - Useful for radars with moderate to wide beamwidths
  - Favorable geometry helpful

- TRADEX and several other radars have multi-target trackers
  - Primary target is kept on boresight with standard monopulse angle tracker
  - Up to 10 other targets, in radar beamwidth, are tracked in range

- Some other radars track in Doppler and in range along with tracking in angle

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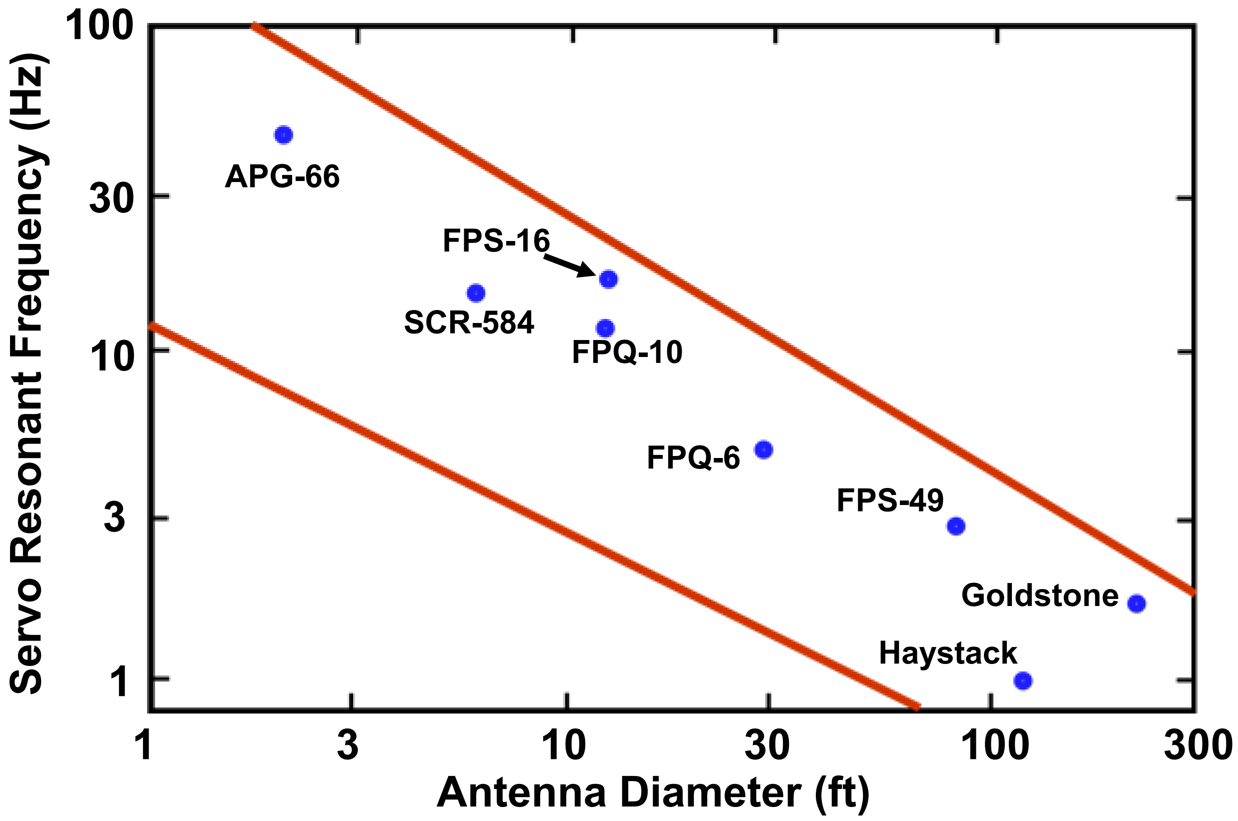
Antenna Servo Systems

- The automatic tracking of a target in angle employs a servo system that utilizes the angle error signals to maintain the pointing of the antenna in the direction of the target
- The servo system introduces lag in the tracking that results in error
  - The lag error depends on the target trajectory
    - Straight line, gradual turn, rapid maneuver
- Type II Servo System often used in tracking radar
  - No steady state error when target velocity constant
  - Known as “zero velocity error system”
- The effect of velocity and acceleration on a servo system can be described by the frequency response of the tracking loop

Servo Bandwidth

- The tracking bandwidth of a servo system is that of a low pass filter
- The bandwidth should be narrow to:
    - Minimize the effects of noise,or jitter,
    - Reject unwanted signal components
        - Conical scan frequency or jet engine modulation
    - Provide a smoothed output of the desired measured parameters
- The bandwidth should be wide to:
    - Follow rapid changes in the target trajectory or in the vehicle carrying the radar
- The choice of servo bandwidth is usually a compromise
    - Sensitivity vs. tracking of maneuvering target
- Tracking bandwidth may be made variable or adaptive
    - Far range - angle rates low, low S/N (narrow bandwidth)
    - Short range - angle rates large (wide bandwidth)
    - Shorter ranges - Glint can be an issue (narrow bandwidth)

Bounds on Servo Resonant Frequency



- The tracking bandwidth of a mechanical tracker should be small compared to the lowest natural frequency of the antenna and its structural foundation
  - This prevents the antenna from oscillating at its resonant frequency

Summary – Part 1

- A detailed description of the different radar observables and their estimation was presented
  - Observables - Range, angle, and Doppler velocity
  - Radar cross section issues were presented in a previous lecture
  - Resolution, precision and accuracy were discussed

- The different techniques for single target angle tracking were discussed in detail, as well as their implementation
  - Amplitude monopulse
  - Phase comparison monopulse
  - Sequential lobing
  - Conical scanning

- Range tracking techniques, as well as other related subjects were presented

Homework Problems

- From Skolnik, Reference 1
  - Problems 4.1, 4.3, 4.5, 4.9, 4.11, and 4.15

References

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2. Barton, D. K., *Modern Radar System Analysis*, Norwood, Mass., Artech House, 1988
3. Skolnik, M., Editor in Chief, *Radar Handbook*, New York, McGraw-Hill, 3rd Ed., 2008
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6. Barton, D. K. and Ward, H. R, *Handbook of Radar Measurements*, Norwood, Mass., Artech House, 1984

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Part 2

- Introduction
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