



# Radar Systems Engineering Lecture 4 The Radar Equation

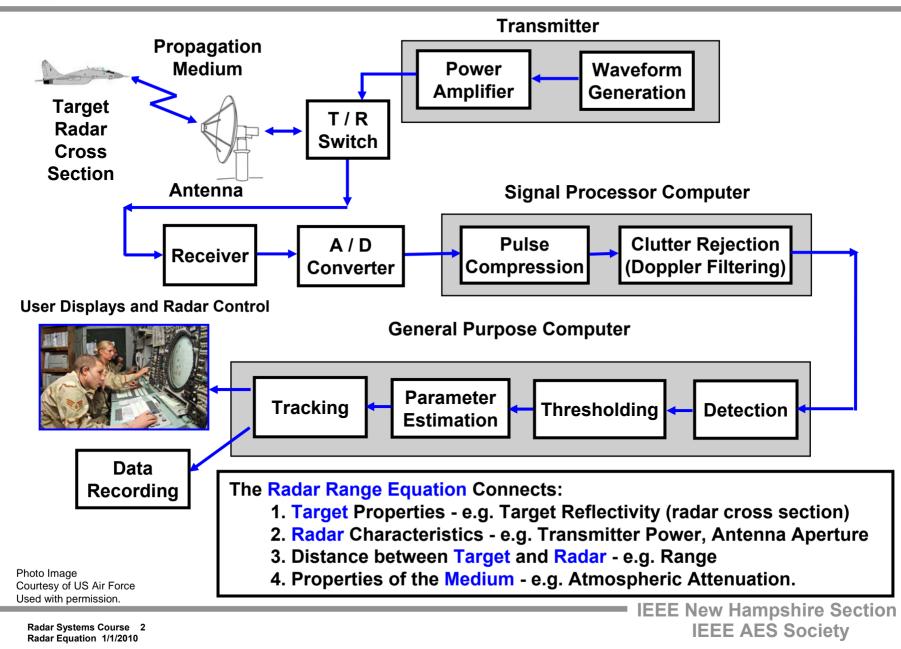
## Dr. Robert M. O'Donnell IEEE New Hampshire Section Guest Lecturer

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- Introduction
- Introduction to Radar Equation
  - Surveillance Form of Radar Equation
  - Radar Equation for Rain Clutter
  - Radar Losses
  - Examples
  - Summary

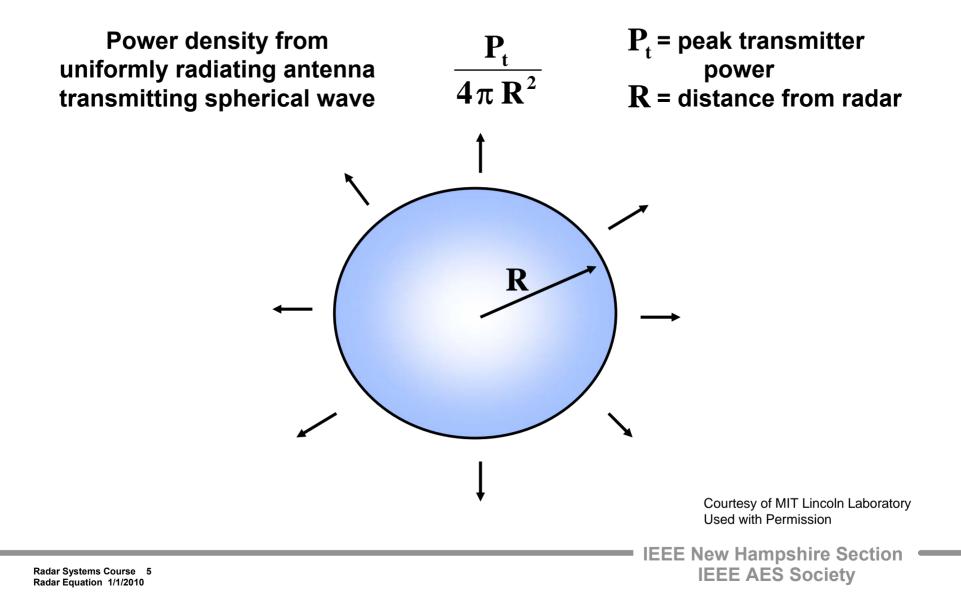




- Detection
  - Illuminate selected area with enough energy to detect targets of interest
- Measure target observables
  - Measure range, Doppler and angular position of detected targets
- Track
  - Correlate successive target detections as coming from same object and refine state vector of target
- Identification
  - Determine what target is Is it a threat ?
- Handover
  - Pass the target on to;
    - Missile interceptor Data Collection function Air Traffic Controller / Operator











Power density from isotropic antenna

Power density from directive antenna

$$\frac{P_t}{4\pi R^2}$$
$$\frac{P_t G_t}{4\pi R^2}$$

- **P**<sub>t</sub> = peak transmitter power
- $\mathbf{R}$  = distance from radar

 $\mathbf{G}_{\mathrm{t}}$  = transmit gain

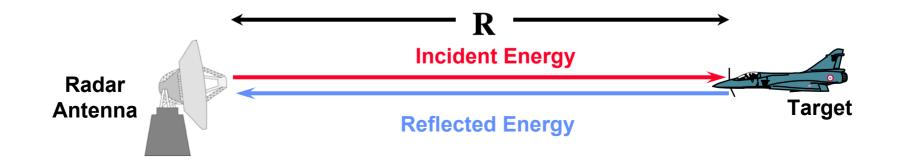
Gain is the radiation intensity of the antenna in a given direction over that of an isotropic (uniformly radiating) source

$$G_t = \frac{4\pi A}{\lambda^2}$$

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# Definition of Radar Cross Section (RCS or s)

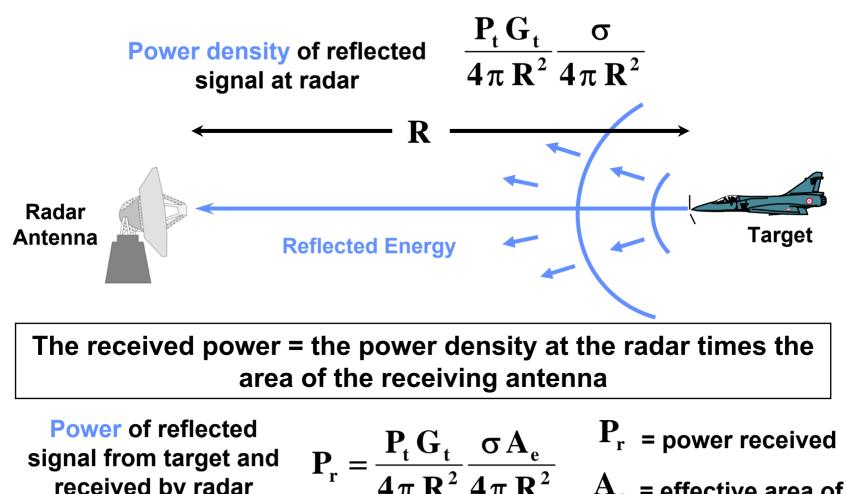


Radar Cross Section (RCS or  $\sigma$ ) is a measure of the energy that a radar target intercepts and scatters back toward the radar

Power of reflected signal at target	$\frac{P_t G_t \sigma}{4\pi R^2}$	<ul> <li>σ = radar cross section units (meters)<sup>2</sup></li> </ul>
Power density of reflected signal at the radar	$\frac{P_t G_t}{4\pi R^2} \frac{\sigma}{4\pi R^2}$	Power density of reflected signal falls off as (1/R <sup>2</sup> )
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 $A_e$  = effective area of receiving antenna

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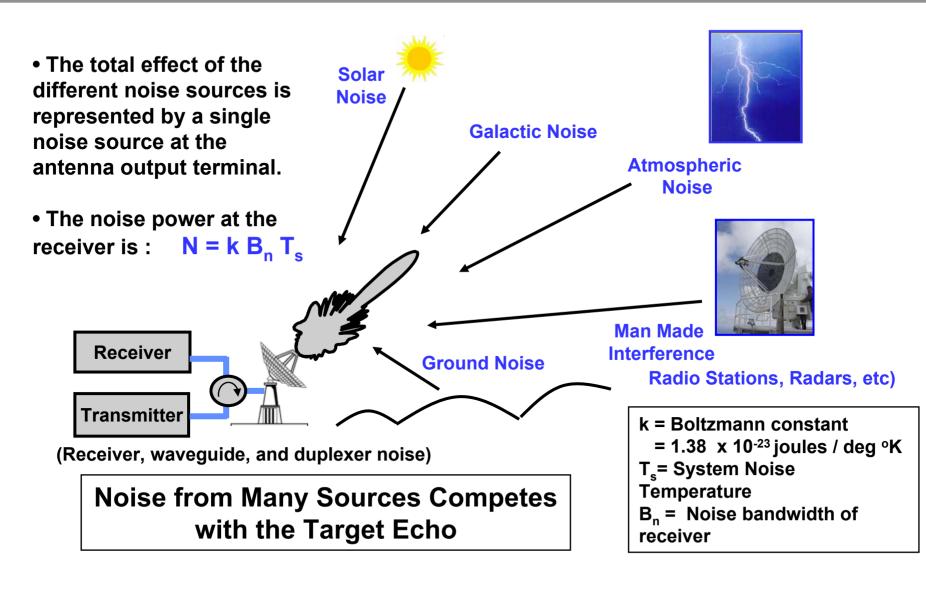
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received by radar

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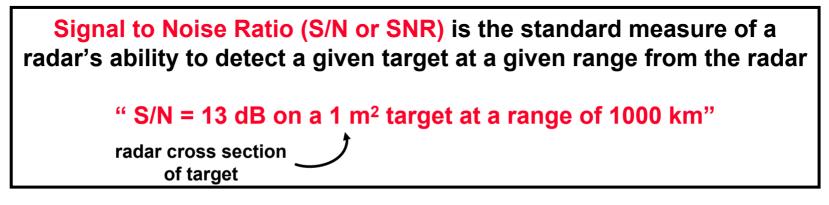


Signal Power reflected from target and received by radar	$\mathbf{P}_{\mathrm{r}} = \frac{\mathbf{P}_{\mathrm{t}} \mathbf{G}_{\mathrm{t}}}{4\pi  \mathbf{R}^2} \frac{\sigma \mathbf{A}_{\mathrm{e}}}{4\pi  \mathbf{R}^2}$	
Average Noise Power	$\mathbf{N} = \mathbf{k} \mathbf{B}_{\mathbf{n}} \mathbf{T}_{\mathbf{s}}$	$\frac{\text{Assumptions}}{\text{G} = \text{G}_{r} = \text{G}_{t}}$ $\text{L} = \text{Total System}$ $\text{Losses}$ $T_{o} = 290^{o} \text{ K}$
Signal to Noise Ratio	$\frac{\mathbf{S}}{\mathbf{N}} = \frac{\mathbf{P}_{\mathbf{r}}}{\mathbf{N}}$	

- 2 - 2

$$\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_s B_n}$$

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The System Noise Temperature,  $T_{\!_{s}}$  ,is divided into 3 components :

Where:

$$\mathbf{T}_{\mathrm{s}} = \mathbf{T}_{\mathrm{a}} + \mathbf{T}_{\mathrm{r}} + \mathbf{L}_{\mathrm{r}}\mathbf{T}_{\mathrm{e}}$$

 $T_{\!_a}$  is the contribution from the antenna

 $T_{\!\rm r}\,$  is the contribution from the RF components

between the antenna and the receiver

 $L_r$  is loss of the input RF components (natural units)

 $T_e$  is temperature of the receiver

The 3 temperature components can be broken down further :

 $T_a = (0.88 T_{sky} - 254) / (L_a + 290)$ 

Where:

 $\mathbf{T}_{sky}$  is the apparent temperature of the sky (from graph)

 $T_{r} = T_{tr} (L_{r} - 1)$  and  $T_{e} = T_{o} (F_{n} - 1)$ 

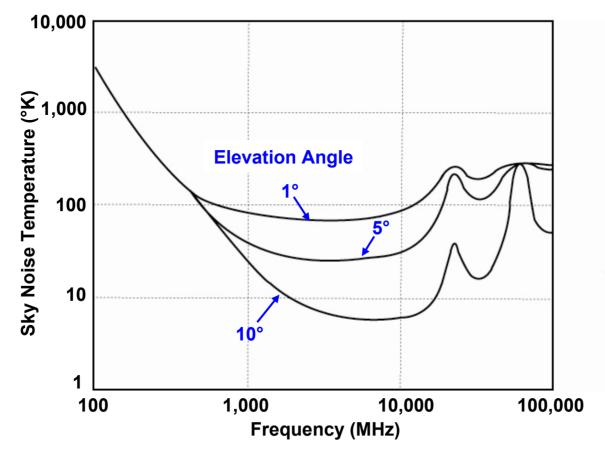
 $\mathbf{L}_{\mathbf{a}}$  is the dissipative loss within the antenna (natural units)

- $\mathbf{T}_{tr}$  is physical temperature of the RF components
- $\mathbf{F}_{n}$  is the noise factor of the receiver (natural units)
- $\mathbf{T}_{0}^{\ddot{}}$  is the reference temperature of 290° K

Note that all temperature quantities are in units of °K







- The data on this graph takes into account the following effects:
  - Galactic noise, cosmic blackbody radiation, solar noise, and atmospheric noise due to the troposphere

(Adapted from Blake, Reference 5, p 170)





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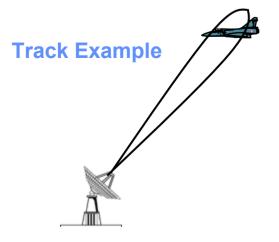




**Track Radar Equation** 

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{\mathrm{P}_{\mathrm{t}} \,\mathrm{G}^{2} \,\lambda^{2} \,\sigma}{(4\pi)^{3} \mathrm{R}^{4} \,\mathrm{k} \,\mathrm{T}_{\mathrm{s}} \,\mathrm{B}_{\mathrm{n}} \,\mathrm{L}}$$

• When the location of a target is known and the antenna is pointed toward the target.

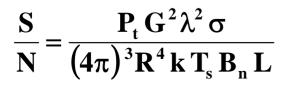


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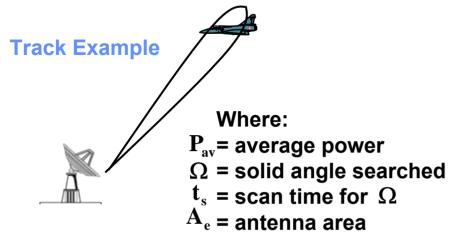




### **Track Radar Equation**



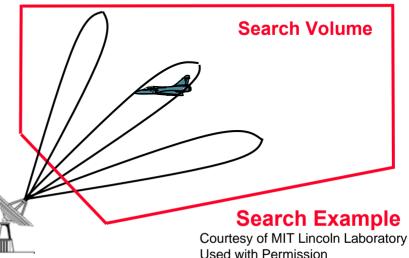
• When the location of a target is known and the antenna is pointed toward the target.



### **Search Radar Equation**

$$\frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4\pi \Omega R^4 k T_s L}$$

• When the target's location is unknown, and the radar has to search a large angular region to find it.



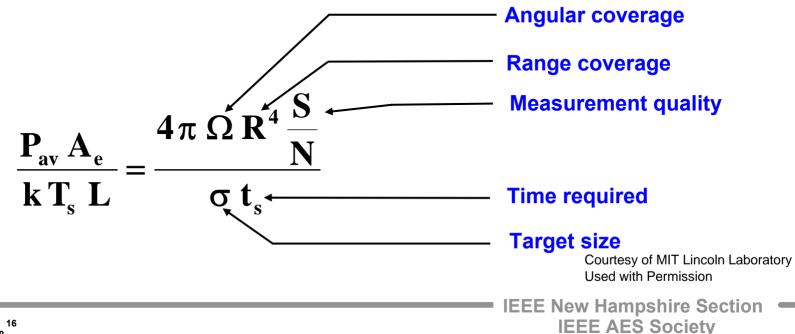




$$\frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4\pi \Omega R^4 k T_s L}$$

**Re-write as:** 

f (design parameters) = g (performance parameters)



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$$\frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4\pi \Omega R^4 k T_s L} \longrightarrow P_{av} = \frac{4\pi R^4 \Omega k T_s L (S/N)}{A_e t_s \sigma}$$

- Power required is:
  - Independent of wavelength
  - A very strong function of R
  - A linear function of everything else

**Example** Radar Can Perform Search at 1000 km Range How Might It Be Modified to Work at 2000 km ?

<u>Solutions</u> Increasing  $\mathbf{R}$  by 3 dB (x 2) Can Be Achieved by:

- 1. Increasing  $P_{av}$  by 12 dB (x 16)
- or 2. Increasing Diameter by 6 dB (  $A_e$  by 12 dB)

An Appropriate Combination of the Above

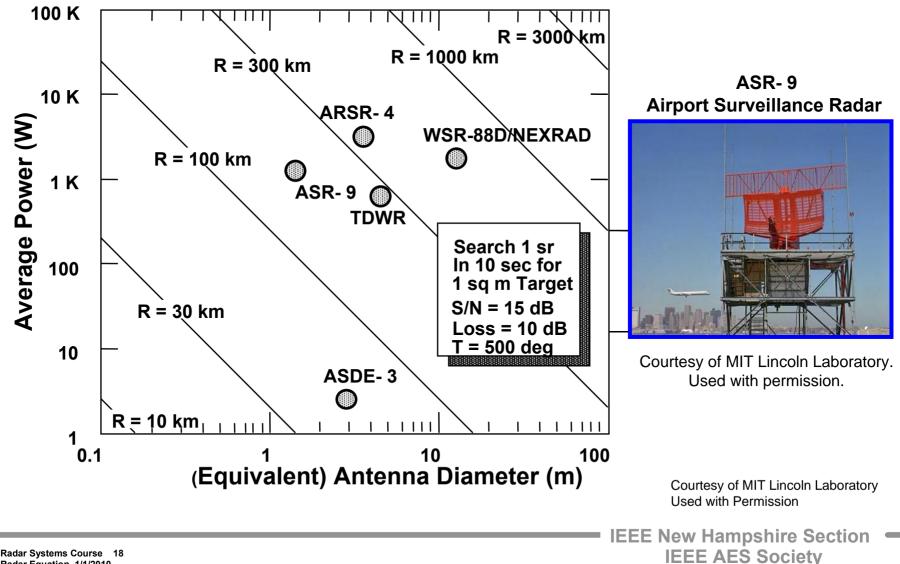
- or 3. Increasing  $t_s$  by 12 dB
- or 4. Decreasing  $\Omega$  by 12 dB
- or 5. Increasing  $\sigma$  by 12 dB

Courtesy of MIT Lincoln Laboratory **Or 6.** Used with Permission

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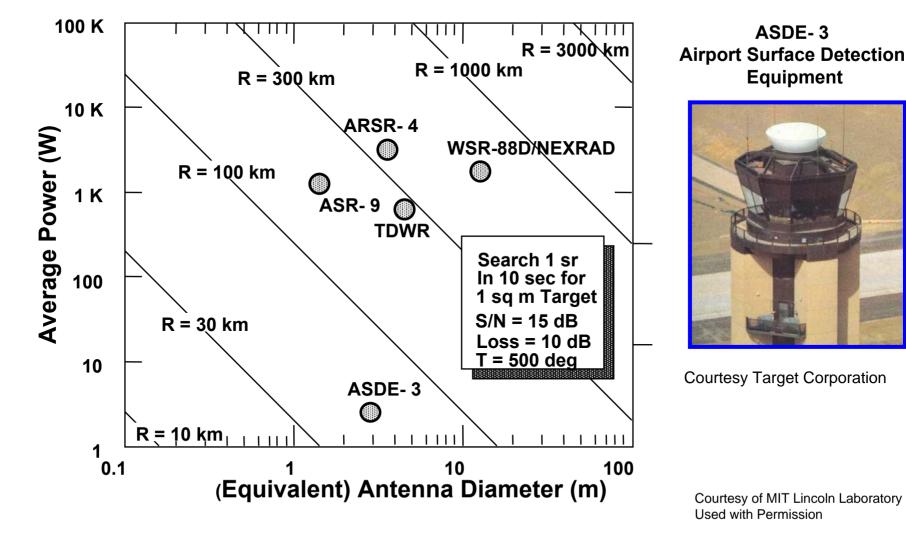






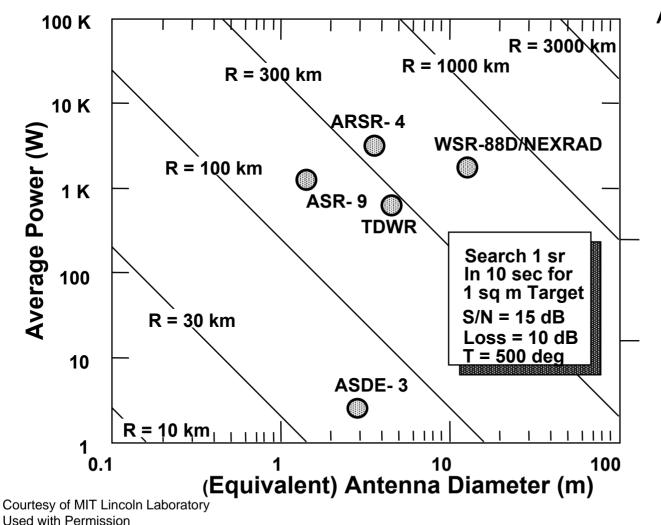












ARSR- 4 Air Route Surveillance Radar



ARSR- 4 Antenna (without Radome)



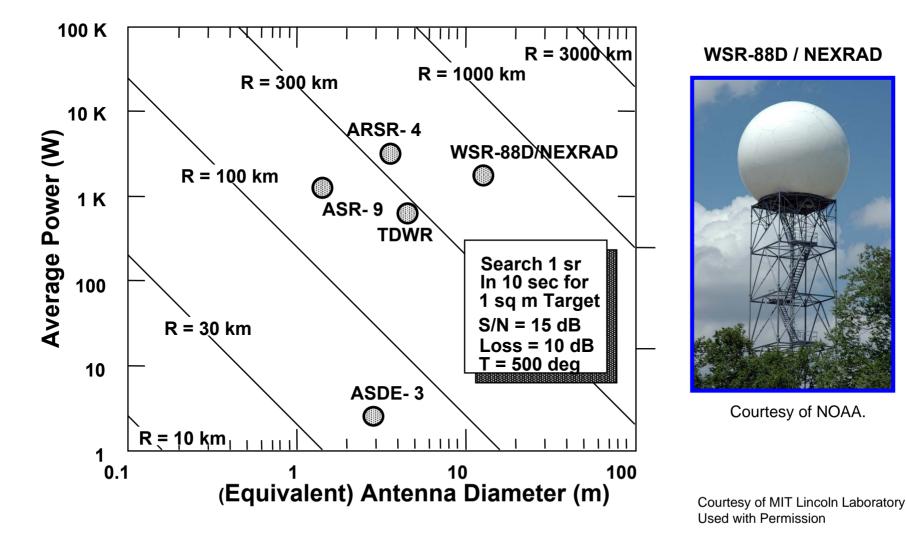
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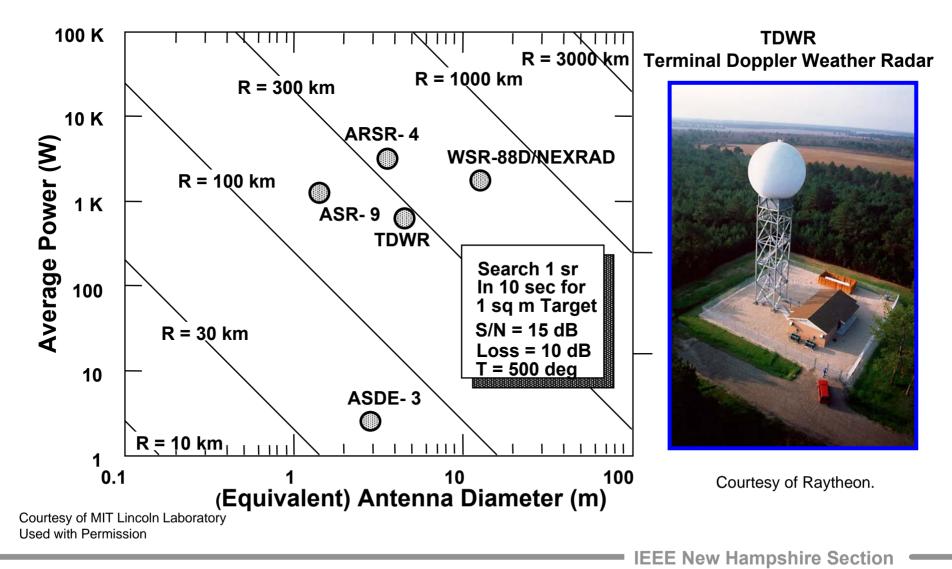








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# Radar Equation for Rain Clutter (and other Volume Distributed Targets)



- Standard radar equation  $\implies \frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_s B_n L}$
- If the target is a diffuse scatterer (e.g. rain), which completely fills the range-azimuth-elevation cell of the radar, then the radar cross section of the target takes the form:

$$\sigma = \eta V$$
 and  $V = \frac{\pi}{4} (R \theta_B) (R \phi_B) \left(\frac{c \tau}{2}\right) \frac{1}{2 \ln_e 2}$ 

• And the radar equation becomes:

$$\frac{S}{N} = \frac{P_t G \lambda^2 c \tau \eta}{1024 (\ln_e 2) R^2 k T_s B_n L}$$
Note, for  
Gaussian  
antenna  
pattern
$$G \approx \frac{\pi^2}{\theta_B \phi_B}$$

• Note, that volume distributed backscatter is a function of  $1/\,R^2$  rather than the usual  $1/\,R^4$ 





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## **Transmit Losses**

Radome Circulator Waveguide Feed Waveguide Antenna Efficiency Beam Shape Low Pass Filters Rotary Joints Scanning Atmospheric Quantization Field Degradation

# **Receive Losses**

Radome Circulator Waveguide Feed Wavequide Combiner **Receiver Protector Rotary Joints** Transmit / Receive Switch Antenna Efficiency **Beam Shape** Scanning **Doppler Straddling Range Straddling** Weighting **Non-Ideal Filter** CFAR Quantization **Atmospheric Field Degradation** 

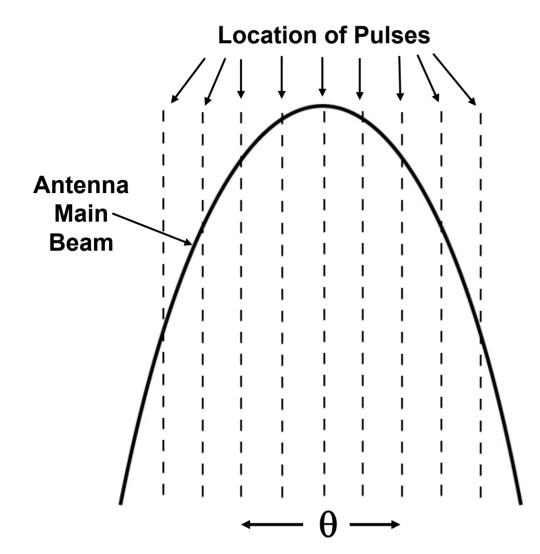




- Beam Shape Loss
  - Radar return from target with scanning radar is modulated by shape of antenna beam as it scans across target. Can be 2 to 4 dB
- Scanning Antenna Loss
  - For phased array antenna, gain of beam less than that on boresite
- Inputs to System Noise Temperature
  - Noise received by antenna
    - Local RF noise
    - Galactic noise
  - Receiver noise factor
  - Receive waveguide losses
  - Antenna ohmic losses







Radar Equation assumes n pulses are integrated, all with gain G.

Except for the pulse at the center of the beam, the actual pulses illuminate the target with a gain less than the maximum.

(Adapted from Skolnik, Reference 1, p 82)





- Waveguide and Microwave Losses
  - Transmit waveguide losses (including feed, etc)
  - Rotary joints, circulator, duplexer
- Signal Processing Loss
  - Range and Doppler Weighting
  - A /D Quantization Losses
  - Adaptive thresholding (CFAR) Loss
  - Range straddling Loss
- Lens Effect Loss
  - Refraction in atmosphere causes spreading of beam and thus degradation in S/N
- Atmospheric Attenuation Loss
  - Attenuation as radar beam travels through atmosphere (2 way loss)





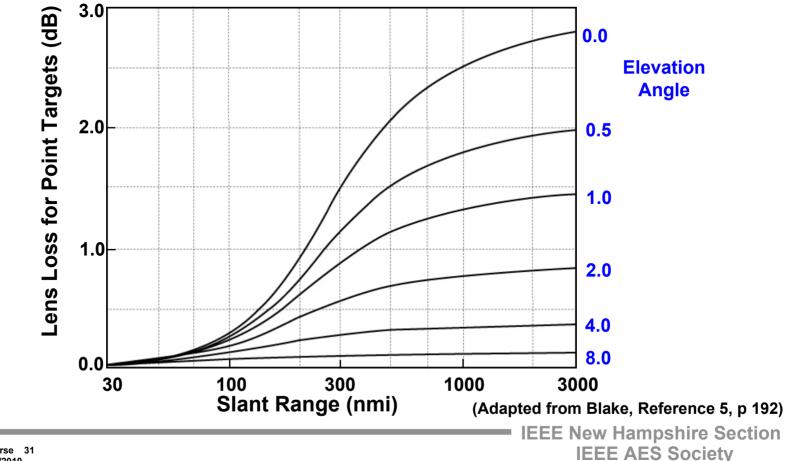
<u>Frequency</u> <u>Band</u>	<u>Frequency Range</u> of Dominant TE <sub>10</sub> Mode (GHz)	<u>Attenuation- Lowest to</u> <u>Highest Frequency (dB/100 ft)</u>
UHF	0.35 - 0.53	0.054 - 0.034
<b>L Band</b> $\int_{1}^{1}$	minum 0.96 - 1.44	0.20 - 0.135
S Band	2.6 - 3.95	1.10 - 0.75
C Band	3.95 - 5.85	2.07 - 1.44
X Band	rass 8.2 - 12.40	6.42 - 4.45
$K_u Band$	12.4 - 18.0	9.58 - 8.04
K <sub>a</sub> Band } cl	ver ad <b>26.5 - 40.0</b> opper	21.9 - 15.0

(Adapted from Volakis, Reference 7, pp 51-40 to 51-41)



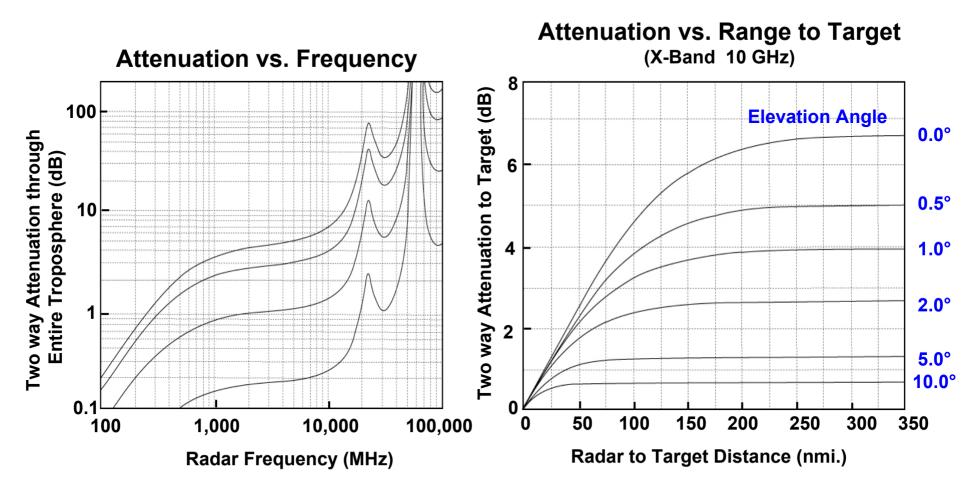


- The gradient of atmospheric refraction at lower elevation angles, causes a spreading of the radar beam, and thus a small diminishment radar power
- This lens loss is frequency independent
- It is significant only for targets that are at long range.









0,1,5,30 deg

(Adapted from Blake, see Reference 5, p 192)





- Bandwidth Correction Factor
  - Receiver not exact matched filter for transmitted pulse
- Integration Loss
  - Non coherent integration of pulses not as efficient as coherent integration
- Fluctuation Loss
  - Target return fluctuates as aspect angle changes relative to radar
- Margin (Field Degradation) Loss
  - Characteristics of radar deteriorates over time (~3 dB not unreasonable to expect over time)

Water in transmission lines

Weak or poorly tuned transmitter tubes

Deterioration in receiver noise figure





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• Summary





- Airport Surveillance Radar
  - 0 th order
  - Back of the envelope

- Range Instrumentation Radar
  - A more detailed calculation





 Problem : Show that a radar with the parameters listed below, will get a reasonable S / N on an small aircraft at 60 nmi.

### Radar Parameters

Range Aircraft cross section Peak Power Duty Cycle Pulsewidth Bandwidth Frequency Antenna Rotation Rare Pulse Repetition Rate Antenna Size

Azimuth Beamwidth System Noise Temp. 60 nmi 1 m<sup>2</sup> **1.4 Megawatts** 0.000525 .6 microseconds 1.67 MHz 2800 MHz 12.7 RPM 1200 Hz 4.9 m wide by 2.7 m high 1.35° 950 ° K

$$\lambda = c/f = .103 m$$
$$G = \frac{4\pi A}{\lambda^2} = 15670$$

= 42 dB, (actually 33 dB with beam shaping losses)

Number of pulses per beamwidth = 21

#### Assume Losses = 8dB

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$$\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_s B_n L}$$

$$P_t = 1.4 \text{ Megawatts} \qquad R = 111,000 \text{ m}$$

$$G = 33 \text{ dB} = 2000 \qquad T_s = 950 \circ \text{K}$$

$$\lambda = .1 \text{ m} \qquad B_n = 1.67 \text{ MHz}$$

$$C = 1 \text{ m}^2 \qquad L = 8 \text{ dB} = 6.3$$

$$k = 1.38 \times 10^{-23} \text{ w} / \text{Hz} \circ \text{K} \qquad (4\pi)^3 = 1984$$

(1.4 x 10<sup>6</sup> w )(2000)(2000)(.1m)(.1m)(1m<sup>2</sup>)

(1984) (1.11 X 10<sup>5</sup> m)<sup>4</sup> (1.38 x 10 <sup>-23</sup> w / Hz ° K) (950 ° K) (6.3) (1.67 x 10<sup>6</sup> Hz)

<u>5.6 x 10+6+3+3-1-1</u> _	5.6 x 10 <sup>+10</sup>	$\frac{5.6 \times 10^{+10}}{-1.05 - 1.0}$
- 415 x 10 <sup>+3+20-23+2+6</sup>	= 4.15 x 10 <sup>+2+3+20-23+2+6</sup>	4.15 x 10 <sup>+10</sup> = 1.35 = 1.3 dB

S / N = 1.3 dB per pulse (21 pulses integrated) => S / N per dwell = 14.5 dB + 13.2 dB Courtesy of MIT Lincoln Laboratory Used with Permission

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dB Method				
		(+)	(-)	
Peak Power	1.4 MW	61.5		
(Gain) <sup>2</sup>	33 db	66		
(Wavelength) <sup>2</sup>	.1 m		20	
Cross section	1 m <sup>2</sup>	0		
$(4\pi)^{3}$	1984		33	
(Range ) <sup>4</sup>	111 km		201.8	
k	1.38 x 10 <sup>-23</sup> w / Hz º K	228.6		
System Temp	950		29.8	
Losses	8 dB		8	
Bandwidth	1.67 MHz		62.2	
		+ 356.1	- 354.8	
		+ 1	.3 dB	

S / N = 1.3 dB per pulse (21 pulses integrated) => S / N per dwell = 14.5 dB (+13.2 dB) Courtesy of MIT Lincoln Laboratory Used with Permission

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 Problem : For a C-band pulsed radar with a 6.5 m dish antenna and 1,000 kW of peak power (0.1% duty cycle), what is the maximum detection range on a target with 0 dBsm cross section, a Pd of .9, and Pfa of 10<sup>-6</sup> (Assume a Swerling Case 1 target fluctuations and a 1 µsec pulse) ?

## Radar Parameters

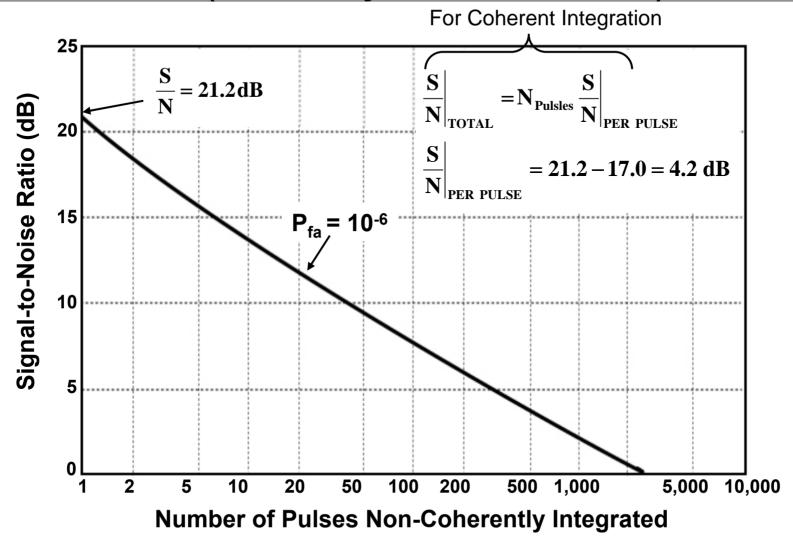
Maximum Detection Range	?? km
Probability of Detection	.9
Probability of False Alarm	<b>10</b> - <sup>6</sup>
Target Cross Section	0 dBsm ( 1 m <sup>2</sup> )
Target Fluctuations	Swerling Case
Peak Power	1,000 Kilowatts
Duty Cycle	0.1 %
Pulsewidth	1 microsecond
Frequency	5,500 MHz
Antenna Size	6.5 m dish
Number of Pulses Integrated	50



## **Detection Statistics for Swerling Case 1**



(Probability of Detection = 0.9)



(Adapted from Blake in Skolnik, see Reference 4, p 192)

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•	Rada	ar and Target Parameters – Inputs	Natural Units	(dB)	
	_	Peak Power (kilowatts)	1,000	60.0	
	_	Pulse Duration (microseconds)	1.0	- 60.0	
	_	Noise Bandwidth (MHz)	1.0	60.0	
	_	Transmit Antenna Gain (dB)		49.6	
	_	Receive Antenna Gain (dB)		49.6	
	_	Frequency (GHz)	5.5		
	_	Wavelength (meters)	5.45	- 25.3	
	_	Single Pulse Signal to Noise Ratio		4.2	
	_	Target Radar Cross Section (meters) <sup>2</sup>	1.0	0.0	
	_	k - Boltzmann's Constant 1.38 x 10 <sup>-23</sup> (w / Hz °K )		- 228.6	
	_	(4π) <sup>3</sup>		33.0	
	_	System Noise Temperature ( °K )	598.2	27.8	
	_	Total System Losses		9.0	
				010	
	_	Range (kilometers)	519		
			Α	ntenna	
			Efficiency	65	%
			Diameter	(meters) <mark>6</mark>	
			Gain (dB)	49.	6
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System Losses Bandwidth Correction Factor (dB) Transmit Loss (dB) Scanning Antenna Pattern Loss (dB) Signal Processing Losses (dB) Atmospheric Attenuation Loss (dB) Lens Effect Loss (dB) Integration Loss (dB) Target Fluctuation Loss (dB) Margin / Field Degradation Loss (dB) Total Loss Budget (dB) Loss – Input to System Noise Temperate Receiver Noise Factor (dB) Antenna Ohmic Loss (dB) Receive Transmission Line loss (dB) Sky Temperature (°K)	(dB) 0.70 1.30 0.00 1.90 1.80 0.25 0.00 0.00 <u>3.00</u> 8.95 ure 4.00 0.20 0.40 50.00	Transmit Losses (dB) Circulator (dB) 0.40 Switches (dB) 0.40 Transmission Line 0.50 1.30Signal Processing Losses (dB) Threshold Loss (dB) 0.50 A/D Quantization Loss (dB) 0.10 Range Straddling Loss 0.20 Weighting Loss 1.10 1.90
Sky Temperature (°K) C-Band at 3° $T_s = T_a + T_r + L_r T_r$		°K
$T_{s} = T_{a} + T_{r} + L_{r} T_{d}$ $T_{a} = (0.88 T_{sky} - 254)$ $T_{r} = T_{tr} (L_{r} - 1)$	$(L_{a} + 2)/(L_{a} + 2)$	290)
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- The radar equation is simple enough, that just about anyone can learn to use and understand
- Unfortunately, the radar equation is complicated enough that anyone can mess it up, particularly if you are not careful
  - See next viewgraph for relevant advice



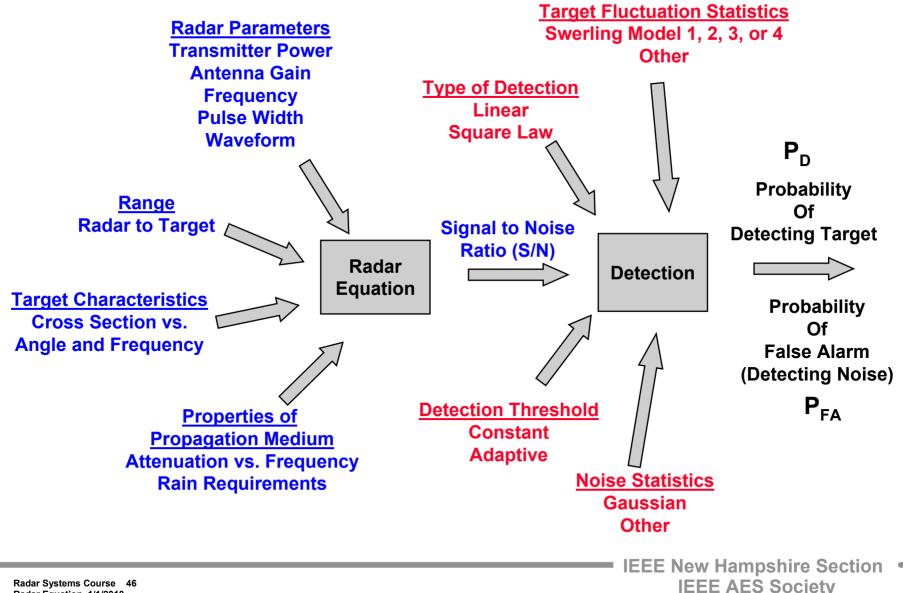
## **The Sanity Check**

- Take a Candidate Radar Equation
- Check it Dimensionally
  - R and  $\lambda$  are distance
  - $-\sigma$  is distance squared
  - $P_t$  is energy / time
  - S/N, G, and L are dimensionless
  - $\ k T_{\!_s} \, \text{is energy}$
  - $\ B_n$  is (time)-1
- Check if Dependencies Make Sense
  - Increasing Range and S/N make requirements tougher
  - Increasing Power and Antenna Gain make radar more capable
  - Decreasing Wavelength and Radar Cross Section make the radar less capable

$$\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_s B_n L}$$

## **Radar Equation and the Detection Process**









- The radar equation relates:
  - Radar performance parameters Detection range, S/N, etc. and
  - Radar design parameters Transmitter power, antenna size, etc.
- There are different forms of the radar equations for different radar functions
  - Search, Track
- Scaling of the radar equation allows the radar designer to understand how the radar parameters may change to accommodate changing requirements
- Be careful, if the radar equation leads to unexpected results
  - Do a sanity check !

Look for hidden variables or constraints

Compare parameters with those of a real radar





- 1. Skolnik, M., *Introduction to Radar Systems*, McGraw-Hill, New York, 3<sup>rd</sup> Ed., 2001
- 2. Barton, D. K., *Modern Radar System Analysis*, Norwood, Mass., Artech House, 1988
- 3. Skolnik, M., Editor in Chief, *Radar Handbook*, New York, McGraw-Hill, 3<sup>rd</sup> Ed., 2008
- 4. Skolnik, M., Editor in Chief, *Radar Handbook*, New York, McGraw-Hill, 2<sup>nd</sup> Ed., 1990
- 5. Blake, L. M., *Radar Range Performance Analysis*, Silver Spring, Maryland, Munro, 1991
- 6. Nathanson, F. E., *Radar Design Principles*, New York, McGraw-Hill, 1<sup>st</sup> Ed., 1991
- 7. Volakis, J. L., *Antenna Engineering Handbook*, McGraw-Hill, New York, 4<sup>th</sup> Ed., 2007





- Dr Stephen D. Weiner
- Dr. Claude F. Noiseux





- From Reference 1, Skolnik, M., Introduction to Radar Systems, 3<sup>rd</sup> Edition, 2001
  - Problem 1-5
  - Problem 1-6
  - Problem 2-22
  - Problem 2-24
  - Problem 2-25